

# **ORP** MODEL DESCRIPTION

## **Optimal Distributions from Tax-Advantaged Retirement Accounts**

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### **Abstract**

The Optimal Retirement Planner (ORP) is a mathematical model that computes an optimal strategic retirement savings plan from a set of user parameters. ORP has been implemented as an Internet based computer application (**i-orp.com**) available for public use. ORP focuses on tax-advantaged and after-tax savings accounts. ORP computes a cash flow that maximizes the amount of money available for spending during retirement. ORP illustrates the power of asset compounding, implications of graduated income taxes and leaving just the right size estate.

The purpose of the model, its Internet implementation, and this paper is to acquaint people new to retirement planning with the issues and terminology of the field.

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## Introduction

Baby Boomers are entering the years just before and just after the year of retirement with little knowledge of the issues and terminology of formulating a strategic retirement plan.

Retirement requires sufficient funds to enjoy a comfortable life while not working for income. For years, the financial press has trumpeted tax-advantage plans as vehicles for funding retirement and with good cause as demonstrated by Keebler [30].

A *tax-advantaged* savings plan is a defined contribution savings plan that is given preferential treatment under the Federal and usually state tax codes.

1. *Tax-deferred* account contributions are made pre taxes, which is to say that such savings are deducted from gross income before taxes are paid. Withdrawals are subject to personal income tax rates. IRAs, 401(k), 403(b), profit sharing, SEP are examples of tax-deferred savings plans.
2. *Tax-exempt* account contributions have had taxes paid on them but withdrawals are made tax free. Roth IRA and the Roth 401(k) are tax-exempt plans.
3. *After-tax Accounts* are conventional savings where taxes are paid contributions and taxes are paid on account returns in the year in which they are incurred.

Appendix B compares Tax-deferred and Roth IRA savings plans.

A strategic retirement plan looks at the broad picture from savings (the accumulation phase) during the employment years through spending of savings during retirement (the distribution phase). A strategic plan considers a couple's individual saving contributions as individuals and retirement cash flow as a couple.

The accumulation phase involves selecting the right kind of tax-advantaged savings plan and budgeting for that saving. The selection is pretty much dictated by factors in the couple's employment situation.

Things get more complicated during the distribution phase when savings are liquidated and spent. The complications are due to IRS regulations and the progressive income tax code that if handled carelessly can decrease funds available for spending.

Well-planned withdrawals from retirement savings will assure that sufficient funds are available for retirement. Three scenarios illustrate what can occur during retirement:

- If withdrawals are made at too high of a rate then the money runs out with the retiree still alive.

- If withdrawals are made at too low of a rate then investment returns will compound and a large estate will remain. Retirement standard of living has been sacrificed to enrich the heirs and tax collector.
- An optimal withdrawal level contains assets that continue to expand early in retirement due to the compounding of investment returns. In the later years, capital is withdrawn and spent. At the end the account balances are brought to the planned estate balance, the amount that is passed to heirs with minimal estate taxes. The retiree, not the heirs and tax collector, enjoys the benefits of retirement savings.

This paper describes a linear programming model that computes a strategic, optimal retirement plan.

- *Strategic* because it covers the entire life cycle, accumulation through distribution.
- *Optimal* because it computes a solution that maximizes the money available for spending during retirement while meeting all of the constraints of retirement.

The ORP model is described in the sections that follow. Example results from the ORP web site (**i-orp.com**) are presented and discussed to shed light on some retirement issues. A glossary is included as Appendix A to help with the understanding the terminology of retirement planning.

The intended ORP audiences are:

- *Younger workers* who need to understand the power of compounding, how it promotes a comfortable retirement, and how important it is to select the right investment plan for their situation.
- *Workers near retirement* who need to understand the nuances of withdrawal techniques.

## Prior Work

Articles in the Journal of the American Association of Individual Investors (AAII) address the problem of distributions from tax-deferred savings. In the June 1995 edition, Hopewell[8] developed the idea of making an estimate of life expectancy and devising a distribution plan that draws down assets to zero over that term. The plan incorporates continued asset growth, taxes, and inflation. In the August 1995 edition, Scott[10] published a Retirement Spending Worksheet for estimating the yearly withdrawals from tax-deferred and after-tax savings, in today's dollars.

More recently scholars, accountants, and financial planners are studying the tax consequences of the choice of which account to withdraw from first during the distribution phase of retirement [19, 20, 21]. The answer to this question usually comes down to "it depends on the situation". This is where computer modeling comes into the picture.

The literature reveals one other linear programming, retirement planning calculator built by Ragsdale et al [23] in 1994, which predates ORP by two years. Ragsdale's formulation takes into account withdrawal limitation ceilings, which have since been eliminated by Congress, as well as minimum distributions from Tax-deferred Accounts. Ragsdale's model requires that the user enter a minimum spending level and the model maximizes spending above the floor and maximizes the estate, but without paying any estate taxes. The computer implementation was used for research but never released to the public.

## **Assumptions**

Strategic retirement planning begins with these precepts:

- You are going to die.
- You can't take your IRA with you.
- Inflation is not going away. Even small rates of inflation erode capital.
- Taxes must be paid.
- Assets grow through the compounding of their returns until they are withdrawn.
- Comfortable retirement requires saving early and saving often.

## **Beyond the Scope of This Paper:**

*Estate Planning:* Every retiree has an actuarial life expectancy. The optimal strategy is to provide for a comfortable lifestyle throughout retirement and to provide a cushion in case the term is exceeded. The exact form of this cushion and how it is transferred to the heirs is estate planning.

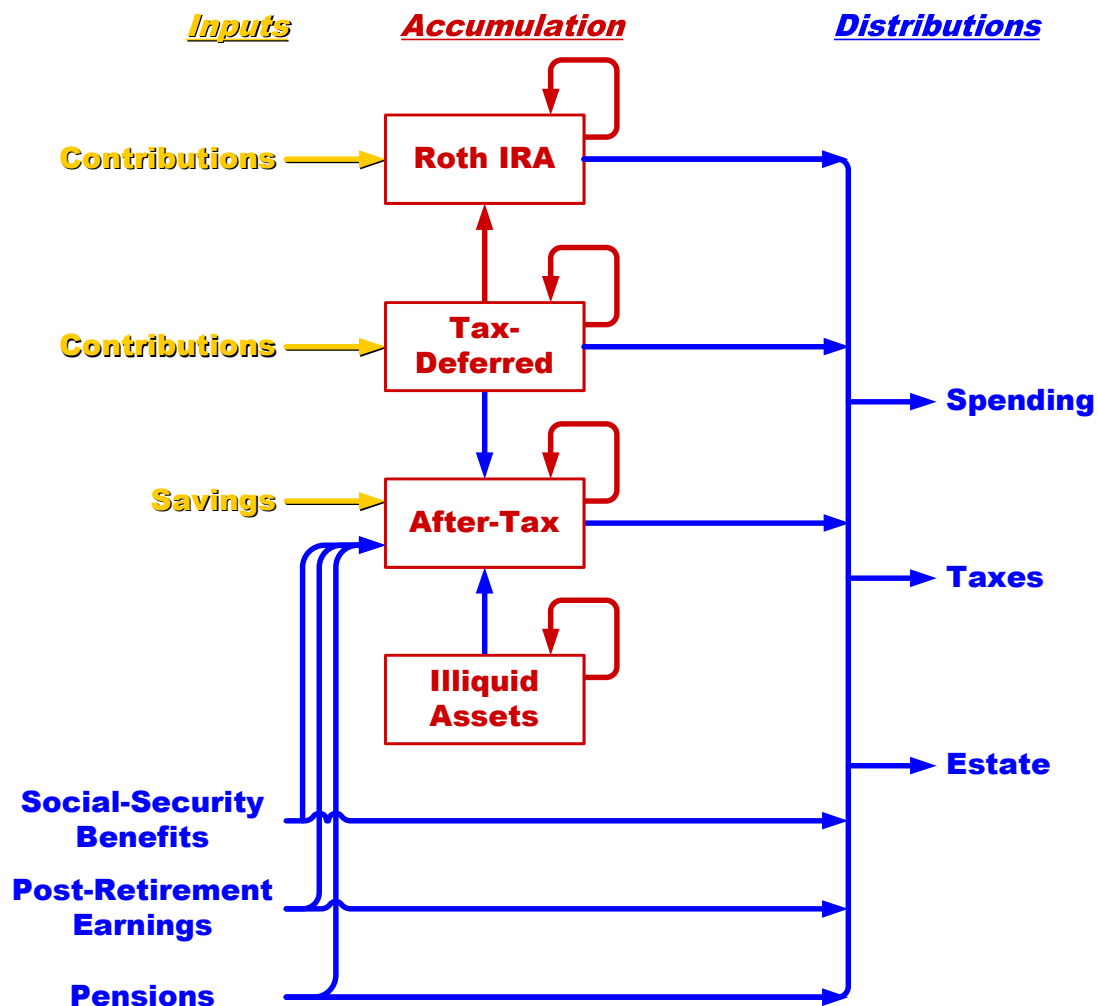
*Investment Strategies:* ORP assumes a fixed, average investment return over the span of the retirement plan. Positive, average returns on investments can be achieved over the years, although there will be yearly fluctuations. Historically, common stocks have returned 10% per year while bonds returned somewhat less [8]. How this average return is achieved is investment strategy.

*Risk Assessment:* ORP makes assumptions about fixing certain parameters throughout the plan. Risk assessment retirement planners [27, 28] use advanced statistical techniques to compute probability distributions about the likelihood of success of an individual's retirement plan. Success is defined as not running out of money too early due to outside factors.

## **Description of ORP**

This section provides an overview of the ORP model. ORP computes an optimal retirement plan that maximizes the total amount of money available for retirement and shows the amount available for living expenses, after inflation and taxes, for each year of retirement.

Figure 1 shows the architecture of the ORP model.



**Figure 1: The ORP Model**

ORP is actually a time dynamic, process flow model with the main feature being inventory of product over time. The product is dollars, both coming in and going out. The dollars are “inventoried” and compounded in savings accounts.

- The *Tax-deferred Account* contains the money invested in one or more IRS approved tax-deferred plans. Money enters the *Tax-deferred Account* from the initial balance, annual pre retirement contributions, and investment return compounding. Contributions to the *Tax-deferred Account* are not taxed. ORP computes and reports the *Tax-deferred Account* balance for each year. The balance will vary by year according to pre-retirement contributions, investment return, and the level of distributions.

- The *Roth IRA* account contains contributions on which taxes have been paid and on which no further federal tax is due. ORP computes and reports the Roth-IRA Account balance for each year. The balance will vary by year according to pre-retirement contributions, investment return, and the level of distributions.
- The *After-tax Account* contains money on which tax has been paid and on which taxes are paid on each year's investment returns. . ORP computes and reports the After-tax Account balance for each year. The annual After-tax Account balance will vary according to pre-retirement contributions, taxes paid, after-tax investment return, transfers from the Tax-deferred Account, and the level of withdrawals.
- *Illiquid Assets*: An illiquid asset cannot be incrementally sold. It can be sold as a unit at some given age. The proceeds from the sale are transferred to the After-Tax Account and distributed annually thereafter. A typical illiquid asset is the retiree's home which, at some point, will be sold in favor of a move to a retirement community. The value of an illiquid asset increases at the rate of inflation.
- *Pre-retirement Earned Income*: Earned income after retirement comes from full or part time employment and which is scheduled to end a few years into the plan. Earned income increases annually at the rate of inflation. Earned income goes directly to spending since it reduces the money drawn from savings.
- *Social Security Benefits*: Social Security Benefits are adjusted for inflation and go directly into spending since they reduce the money drawn from savings.
- *Pensions*: Pension income may be subjected to inflation increases or not depending on the pension plan. ORP models both types. Pension income goes directly to spending since it reduces the money drawn from savings.

Money is withdrawn from any or all accounts to meet living expenses and to contribute to the estate requirements. Beginning at age 70 ½, the IRS requires a minimum distribution from the Tax-deferred Account. In some cases money from the Tax-deferred Account may transferred to the After-tax Account, to meet IRS requirements for making minimal distribution and the money is being saved for a relatively large estate. Tax-deferred withdrawals are taxed at ordinary income tax rates.

ORP maximizes “Spending”, the sum of the withdrawals from all accounts after taxes and penalties have been paid over time.

Process flow models have been around since the dawn of linear programming 60 years ago [29]. It is usually straightforward to reduce such a model to a set of linear equations and from there solving and reporting the model is done by commercial or open source software.

## ORP Model Parameters

The core parameters that drive ORP are shown in Figure 2.

Age	Dollars
Current Age	Beginning Account Balances
	Account Contributions
Retirement Age	
Life Expectancy	Desired Estate

**Figure 2: Core Parameters**

There are other ORP parameters as shown later in Figure 3 below. Even more are on ORP's web site ([i-orp.com](http://i-orp.com)) input form and they are explained by the help files therein.

ORP results begin with a summary report which is supported by several detail reports that shows the model's results over time. These reports are illustrated in the next section.

## Computational Results

The demonstration of ORP's view on some of the issues of retirement is presented in a scenario structure. It has long been accepted in the linear programming world that the results of a single model are interesting but just the starting point [24]. That single result is the base scenario. A model becomes more interesting when a number of additional scenarios are developed to explore the sensitivity of the model to parameter variations. In what follows a base scenario is established and nine alternative scenarios are investigated.

### Base Scenario

The Base Scenario is a retirement plan for a married couple, John and Jane Doe. Figure 3 shows the basic set of parameters to describe the Doe's retirement situation. Dollar amounts are in thousands of dollars as is generally true throughout this paper.

<i>Description</i>	<i>Jon Doe</i>	<i>Jane Doe</i>	<i>Discussion</i>
Current Age	60	57	The Does are not yet retired
Balance of Tax-deferred Accounts (in \$000)	900	500	Both are employed and participating in their companys' 401(k) plans.
Planned Yearly Contributions to Tax-deferred Accounts	7	3	401(k) annual contributions
Balance of After-tax Account	50		The After-tax account is shared as a family and not assigned to individuals
Planned Yearly Contributions to	2		Savings outside of tax-advantaged accounts

After-tax Account			
Desired Estate Size	10		Subject to inflation
Anticipated Retirement Age	65	65	Jane will continue working for three years after John retires.
Social Security Benefits	21	21	Typical of a working couple
Age at Which the Plan is to End	92		IRS determined life expectancy of a least one partner of a couple.

**Figure 3: Base Scenario Parameters**

ORP supports several more parameters which are not needed for this discussion but are available on ORP's web parameter page and explained in the ORP help document. For the sake of the base scenario these other parameters are set at their default values. All values in Figure 3 are in today's dollars.

The Does chose to use tax-deferred accounts for their retirement saving because has been conclusively shown that these accounts produce larger valuations than does an after-tax account [30].

Table 1 summarizes the ORP solution to the Base Scenario.

Annual After-tax Spending(in today's dollars):	\$163,300
Account Balances at Retirement	2,176,600
Total Plan Value	8,984,400
Estate	30,000

**Table 1: Base Scenario Results**

ORP's results are that this couple will have \$163,300 to spend annually in current, after-tax, dollars. The Does' account balances at retirement (John's age 65) will total \$2,176,600 in inflated dollars 5 years from now. The total dollars to be withdrawn will be \$8,984,400, after inflation. While the specified estate was \$10,000 the value of their estate will be \$30,000 in inflated dollars.

ORP supports this plan's summary with additional detailed reports.

Table 2 shows how the Doe's asset balances compound and then are spent down over the life of their plan..

<b>Age</b>	<b>Tax-Deferred</b>	<b>After-Tax</b>	<b>Total</b>
060	1400	50	1450
061	1550	56	1606
062	1715	63	1778
063	1898	70	1967
064	2098	77	2176
065	2261	13	2274
066	2339	4	2343
067	2411	0	2411
068	2507	0	2507
069	2546	0	2546
070	2581	0	2581
071	2612	0	2612
072	2639	0	2639
073	2659	0	2659
074	2674	0	2674
075	2681	0	2681
076	2679	0	2679
077	2668	0	2668
078	2646	0	2646
079	2612	0	2612
080	2564	0	2564
081	2500	0	2500
082	2418	0	2418
083	2317	0	2317
084	2194	0	2194
085	2046	0	2046
086	1870	0	1870
087	1664	0	1664
088	1423	0	1423
089	1144	0	1144
090	822	0	822
091	452	0	452
LastYear	30	0	30

**Table 2: Asset Balance Report**

The first column is John's age since he is considered the plan's retiree.

The second column is the balance of the Tax-deferred Account at the end of each year. All account balances in these reports are as of the end of the year. See Appendix C for discussion of ORP's account arithmetic.

Of particular interest is that the After-tax Account (Column 3) is liquidated during the first years of John's retirement.

The Tax-Deferred account continues to grow for a full 10 years until John is 75 years old then it begins contracting. The lesson here is that in a well formulated retirement plan the

assets on hand in the year of retirement are only part of what will be available for spending and taxes during retirement thanks to compounding of investment returns.

Table 3 shows retirement cash flow.

Age	Tax-Deferred	After-Tax	SocSec	Earnings	Taxes	Spending
065	59	73	23	53	14	194
066	147	11	24	55	37	201
067	162	4	25	57	40	208
068	145	0	52	59	41	215
069	212	0	53	0	43	223
070	219	0	55	0	44	230
071	227	0	57	0	46	239
072	235	0	59	0	47	247
073	243	0	61	0	49	256
074	252	0	63	0	51	264
075	260	0	66	0	52	274
076	270	0	68	0	54	283
077	279	0	70	0	56	293
078	289	0	73	0	58	304
079	299	0	75	0	60	314
080	309	0	78	0	62	325
081	320	0	81	0	64	337
082	331	0	84	0	67	348
083	343	0	86	0	69	360
084	355	0	90	0	71	373
085	367	0	93	0	74	386
086	380	0	96	0	76	400
087	394	0	99	0	79	414
088	407	0	103	0	82	428
089	422	0	106	0	85	443
090	436	0	110	0	88	459
091	452	0	114	0	91	475
LastYear	467	0	118	0	94	491

**Table 3: Withdrawal and Spending Report**

The Spending (the rightmost) column shows the amount of money, after inflation, available for spending each year of the plan. Spending is the maximum withdrawal level computed by ORP, including adjustments for inflation and after taxes.

The amounts shown in the Tax-Deferred column of Table 3 are withdrawals from the Tax-deferred account before taxes have been paid. At retirement, the After-tax Account is spent first but withdrawals also begins from the Tax-deferred Account at the same time. This is contrary to the advice of professional financial advisors [1]. The source of the anomaly is being studied by the academic and financial communities [19, 20]. The cause is the graduated Federal income tax code. Since personal income tax has to be paid on tax-deferred distributions it turns out to be better to take early, relatively small distributions from the Tax-deferred Account since they are taxed in the lower 10% and 15% tax brackets. These relatively small withdrawals are in parallel with After-tax Account withdrawals (or in some cases parallel with Roth IRA withdrawals). Delaying Tax-deferred Account withdrawals until later will push them into the 25% tax bracket or above. The alternative is to take all Tax-deferred Account withdrawals later in the plan

to take maximum advantage of tax deferred compounding. In this case ORP resolves the issue in favor of the former, parallel withdrawal strategy.

The SocSec column is Social Security benefits received. In the first 3 years of the plan only John is collecting Social Security. During that same period Jane is still getting earned income, but no Social Security. The Taxes column shows Federal Income Taxes due. The Base Scenario assumes that the Does live in a state that has no state income tax, such as Nevada or Texas. In Table 3 the distributions from the Tax-deferred Account continue to increase according to inflation whereas in Table 1 the balance in the Tax-Deferred Account balance rose and then fell over the life of the plan.

Table 4 shows the activity in the Tax-Deferred Account for each year of the plan.

Age	Account Bal	Contrib	Distrib	MinDist
060	1400	10	0	0
061	1550	10	0	0
062	1715	11	0	0
063	1898	11	0	0
064	2098	11	0	0
065	2261	0	59	0
066	2339	0	147	0
067	2411	0	162	0
068	2507	0	145	0
069	2546	0	212	0
070	2581	0	219	87
071	2612	0	227	99
072	2639	0	235	102
073	2659	0	243	122
074	2674	0	252	127
075	2681	0	260	133
076	2679	0	270	138
077	2668	0	279	144
078	2646	0	289	149
079	2612	0	299	153
080	2564	0	309	157
081	2500	0	320	160
082	2418	0	331	161
083	2317	0	343	161
084	2194	0	355	163
085	2046	0	367	160
086	1870	0	380	153
087	1664	0	394	148
088	1423	0	407	133
089	1144	0	422	113
090	822	0	436	86
091	452	0	452	50
LastYear	30	0	467	4

**Table 4: Tax-Deferred Account Activity Report**

The ages shown in Table 4 are John's age. All values are in thousands of dollars. For a discussion of how ORP arithmetic is done and its activities are timed see Appendix C.

The balance in the Tax-deferred Account continues to compound until John is 75 at which point capital drawdown begins.

Contributions cease when John retires at age 65. It really doesn't make much sense for Jane to be contributing, although legally she could, while John is withdrawing. The far right column shows the minimum required distribution kicking in when John reaches the age of 70 ½. These values are uneven because they are computed from the Tax-deferred Account balance, which rises and falls but as time goes by the life expectancy value in the computation keeps getting extended into the future.

Table 5 shows the first part of the After-tax Account Report for the married couple scenario.

Age	Acc Bal	Savings	Distrib
060	50	2	0
061	56	2	0
062	63	2	0
063	70	2	0
064	77	2	0
065	13	0	73
066	4	0	11
067	0	0	4
068	0	0	0
069	0	0	0
070	0	0	0

**Table 5: After-Tax Account Activity Report**

The After-Tax Account balance increases through John's age of 64 due to compounding and savings. Beginning at age 65, the account is emptied over the course of the next three years.

Table 6 shows the Doe's taxable income broken down by the Federal income tax bracket.

<b>Age</b>	<b>0-NoTax</b>	<b>10%</b>	<b>15%</b>	<b>25%</b>
065	30	18	85	0
066	31	19	88	86
067	32	19	91	99
068	34	20	95	100
069	35	21	98	103
070	36	21	102	107
071	38	22	105	111
072	39	23	109	114
073	40	24	113	118
074	42	24	117	123
075	43	25	121	127
076	45	26	125	131
077	46	27	129	136
078	48	28	134	141
079	50	29	139	146
080	51	30	143	151
081	53	31	148	156
082	55	32	154	161
083	57	33	159	167
084	59	34	165	173
085	61	36	170	179
086	63	37	176	185
087	65	38	182	192
088	68	40	189	199
089	70	41	195	205
090	73	42	202	213
091	75	44	209	220
Last Year	78	45	217	228

**Table 6: Income by Federal Tax Bracket.**

The rows of Table 6 are identified by John's age. The columns are tax brackets taking into account the Does' standard deduction.

The relatively small Tax-Deferred distribution taken in the first year of John's retirement is taxed up to 15%. Thereafter the 25% bracket applies. Taxes paid in the early years are fairly uniform even though John goes on Social Security while Jane continues to work. When John is 68 Jane retires and begins taking Social Security. In point of fact it is ORP's purpose to keep income and thus taxes uniform. There are situations when there will be step changes in taxes paid in portfolios with large After-Tax savings or at the disposition of an illiquid asset.

Due to inflation the taxable amounts are rising each year but the next higher tax bracket is never reached because the bracket thresholds are being inflated at the same time.

## Scenario 1: Lump-sum Distribution

The first scenario discusses the intuitively appealing retirement strategy of making a lump-sum distribution from the Tax-Deferred at retirement and managing it solely as an After-Tax Account. The Base Scenario parameters are as taken from Figure 3, and modified with three changes:

1. Both John's and Jane's Tax-deferred Accounts are liquidated, taxes are paid and the remaining balance is invested in the After-tax Account.
2. There is no beginning Tax-deferred Account balance.
3. The After-tax Account's beginning balance is the sum of the After-tax Account balance at age 60 and the Tax-Deferred Account at age 60 after personal income taxes have been paid. Both values come from Table 1 of the Base Scenario reports.
4. The Tax-deferred Account contributions are switched to the After-tax Account after taxes have been paid.

Table 7 compares the difference between the Base and Lump-sum scenarios.

	<i>Base</i>	<i>Lump-sum</i>
<i>Tax-Deferred Beginning Balance</i>	\$1,400,000	\$0
<i>After-Tax Beginning Balance</i>	50,000	988,000
<i>Withdrawal Level</i>	163,300	129,900
<i>Plan Value</i>	8,984,000	7,070,000

**Table 7: Lump-sum Distribution**

These results are definitely not in favor of the Lump-sum distribution approach. They do demonstrate the superiority of tax advantaged savings over conventional savings.

## Scenario 2: Impact of Inflation

The anticipated average rate of inflation is an important assumption for ORP. Although the inflation rate from 1985 to 2008 has been under 3% and was 2.6 in 2006 [22], the inflation rate from 1980 through 1991 averaged 4.66% per year [8]. Table 8 demonstrates the effect of different inflation rates on the withdrawal level of the model. All parameters are the values shown in Figure 3 of the Base Scenario. The Withdrawal values shown are the annual after-tax spending amounts as measured in today's dollars. The value column contains the total amount of money available throughout retirement. The Base Scenario inflation rate was 3.5%

<i>Inflation % Rate</i>	<i>Withdrawal</i>	<i>Total Plan Value</i>
2.6	\$178,800	8,196,600
3.0	171,100	8,534,400
3.5	163,300	8,964,400
4.0	156,600	9,456,600
4.5	148,800	9,965,500
5.0	141,100	10,516,600
7.0	117,700	13,218,800
10.0	88,000	19,011,100

**Table 8: The Effects of Inflation**

Inflation effects the levels of after-tax withdrawal as measured in current dollars. The higher the inflation rate the less inflation-adjusted money there is to spend each year. There is no change in how the retirement plan is managed, only in the annual withdrawal level. The total value of the retirement plan goes up as inflation increases, but it buys fewer goods and services.

In The Age of Turbulence former Federal Reserve Chairman Alan Greenspan [22] projects a future U.S. inflation rate markedly above the 2006 rate of 2.6%. Greenspan takes a whole chapter to develop why.

The rate of inflation is not something that the retiree can control. Let's just hope that the Federal Reserve remains independent of the political side of the Federal government and the former remains populated with inflation hawks.

### Scenario 3: Adjusting the Investment Return Rate

The expected rate return on investments in all accounts also has a significant impact on the withdrawal level as measured in today's dollars. These results are shown in Table 9. All parameters are the values shown in Figure 3 except for variations in the rate of investment return. The Base Scenario uses a 10% portfolio rate of return.

<i>% Rate of Return</i>	<i>Withdrawal Level</i>
4	\$93,000
5	102,200
6	113,300
7	124,400
8	136,600
9	149,900
<b>10</b>	163,300
11	179,900
12	195,500

**Table 9: The Effects of Rate of Investment Return**

The portfolio return is the reward for taking risk. A 4% return is a risk free government bond. A 12% return is a highly aggressive equity portfolio

### Scenario 4: Interacting Inflation and Portfolio Return

The next question to consider is: "Can portfolio return counteract inflation?"

Table 10 shows the annual, after tax, money available for spending for varying levels of inflation and portfolio return. The risk factor is not modeled.

<b>Return %</b>	<b>3% Inflation</b>	<b>4% Inflation</b>	<b>5% Inflation</b>
6	\$118,800	\$107,700	\$97,000
7	130,000	118,800	107,700
8	142,200	129,900	117,700
9	156,600	142,200	129,900
10	171,100	156,600	141,100

**Table 10: Spending for Interacting Inflation and Portfolio Return**

Table 10 shows the answer is “yes” provided that the too much risk is not undertaken in the pursuit of higher returns.

### Scenario 5: Rollover to a Roth IRA

The Roth IRA is an alternative tax-advantaged, retirement saving plan. Since taxes are paid before contributions but no taxes are paid on withdrawals from Roth IRA investments the Roth IRA falls somewhere between a Tax-deferred Account and an After-tax Account. Roth IRA taxes are paid up front. With a Tax-Deferred Account the taxes are delayed until the funds are withdrawn.

In Scenario 1 the effect of taking a lump sum distribution from the After-Tax Account where the taxes paid as incurred was considered, as shown earlier in Table 7. Table 11 shows a scenario where 10% of the Doe’s Tax-deferred Account is rolled over into Roth IRAs, assuming that the Does can meet the restrictions for making such a rollover [11].

	<b>Base</b>	<b>Lump-sum</b>	<b>10% Rollover</b>
<i>Tax-Deferred Beginning Balance</i>	\$1,400,000	\$0	\$1,861,000
<i>After-Tax Beginning Balance</i>	50,000	988,000	50,000
<i>Roth IRA Beginning Balance</i>			139,000
<i>Withdrawal Level</i>	163,300	129,900	162,200
<i>Plan Value</i>	8,984,000	7,070,000	8,925,500

**Table 11: Roth IRA Rollover**

Like the lump-sum scenario personal income taxes of 33% have to be paid of the amount withdrawn from the Tax-deferred Accounts. Unlike the lump-sum scenario no further taxes are due. Even through the Does have rolled over a portion of their Tax-deferred Account they continue to contribute to it until retirement.

Table 12 shows the start of the Withdrawal Report for the Roth IRA rollover.

Age	Tax-Def	After-Tax	Soc Sec	Earnings	Roth IRA	Taxes	Spending
065	59	71	23	53	0	14	193
066	61	16	24	55	58	15	200
067	63	0	25	57	77	16	207
068	81	0	52	59	47	25	214
069	210	0	53	0	0	42	221
070	217	0	55	0	0	44	229
071	225	0	57	0	0	45	237
072	233	0	59	0	0	47	245
073	241	0	61	0	0	48	254

Table 12: Roth IRA Withdrawal Report

The withdrawal plan for the 10% rollover option shows that the After-Tax Account is withdrawn first and the Roth IRA second. Reduced levels of After-Tax Account withdrawals accompany this, again to take advantage of the graduated income tax.

Table 12a shows how the tax brackets that the withdrawals fall into.

PostRetm	0-NoTax	10%	15%	25%
065	30	18	85	0
066	31	19	88	0
067	32	19	91	0
068	34	20	95	35
069	35	21	98	101
070	36	21	102	105
071	38	22	105	109
072	39	23	109	112
073	40	24	113	116
074	42	24	117	120

Table 12a: Tax Schedule for 10% Roth IRA Rollover

Taxes are paid in the 15% bracket until the non Tax-deferred Accounts run out and then withdrawals from the Tax-deferred Account only move into the 25% bracket. In any case the Does are better off leaving their money in their Tax-deferred Accounts.

## Scenario 6: Can John Retire at 55?

For the sake of this scenario let's back up 20 years in time and set John's age to 40 and Jane's to 37. Corresponding reductions are made in Tax-Deferred Account balances, putting John's at \$113,000 and Jane's at \$65,000. The After-Tax Account is reduced to \$6,000. These new amounts reflect reduced values due to contributions and investment returns' compounding that have not been made yet..

Table 13 compares the Base Scenario to two Early Retirement Scenarios. The first is where Jane retires at the same time as John does and the second has Jane working to the age of 65.

	<i>Base Scenario</i>	<i>Jane Retires</i>	<i>Jane Keeps Working</i>
<i>Annual After-tax Spending(in today's dollars):</i>	\$163,300	\$59,000	\$72,000
<i>Account Balances at Retirement</i>	2,176,600	1,087,700	1,087,700
<i>Total Plan Value</i>	8,984,400	7,656,600	9,333,300

**Table 13: John Retires at Age 55**

Even though John retired at 55 he does not take Social Security until age 65.

### Scenario 7: How Much Should I Be Saving?

John gets into his head that \$163,300 is insufficient for retirement and he wants to know how much he can improve the situation by increasing after-tax savings to \$20,000 annually. Jane, who keeps the household books, wants to improve cash flow by eliminating after-tax savings. Table 14 summarizes these two cases.

	<i>Base Scenario</i>	<i>Reduced Savings</i>	<i>Increased Savings</i>
<i>Annual After-tax Spending(in today's dollars):</i>	\$163,300	\$163,300	\$170,000
<i>Account Balances at Retirement</i>	2,176,600	2,166,600	2,261,100
<i>Total Plan Value</i>	8,984,400	8,942,200	9,343,300

**Table 14: Changes in Savings**

Jane's plan to stop after tax savings has very little impact on the Does' retirement. On the other hand John's desire to increase funds available for retirement by increasing after tax savings to \$20,000 will have some impact but not in proportion to the amount of increased savings. This is because there are only five years left until retirement and the After-Tax Account is liquidated first and thus there is very little asset compounding.

## Scenario 8: Living to a Hundred

Some retirees may feel, for a variety of reasons, that they have a reasonable probability of outliving the IRS life expectancy estimate or at least afraid that they might. Table 13 shows the results of setting the plan's term to 100 years.

	<i>Base</i>	<i>Age 100</i>
<i>Withdrawal Level</i>	163,300	154,500
<i>Plan Value</i>	8,984,000	12,824,400

**Table 13: Living to the Age of 100**

The extra 8 years of comfort buffer doesn't cost that much in spending but thanks to inflation it shows significantly larger, total plan resources.

## Scenario 9: Changes in Starting Social Security

Starting Social Security benefits at age 62 is attractive to people who want their money now. Other considers waiting until age 70 to enjoy maximum annual benefits. In Table 14 both of these options are summarized.

	<i>Base</i>	<i>Age 62</i>	<i>Age 70</i>
<i>Annual Social Security Benefits</i> [25]	\$21,000	\$16,500	\$30,000
<i>Annual After-tax Spending(in today's dollars):</i>	163,300	130,000	156,600
<i>Account Balances at Retirement</i>	2,176,600	1,606,600	2,176,600
<i>Total Plan Value</i>	8,984,400	7,592,200	8,599,900

**Table 14: Starting Social Security Benefits**

In both of these scenarios Jane will work up to age 65. In the Age 62 scenario John will retire when Social Security Benefits begin. In the Age 70 scenario John will retire at age 65 and both will forego Social Security benefits until age 70. Both of these options indicate that Social Security benefits are biased against retirement at ages other than 65.

Table 15 shows the beginning of the Withdrawal Report for the Age 70 scenario.

Age	Tax-Def	After-Tax	Soc-Sec	Earnings	Taxes	Spending
065	79	68	0	53	14	186
066	149	20	0	55	32	192
067	181	0	0	57	40	199
068	187	0	0	59	41	206
069	255	0	0	0	42	213
070	232	0	31	0	42	221
071	240	0	32	0	44	228
072	248	0	33	0	45	236
073	221	0	68	0	45	245
074	229	0	71	0	46	253

**Table 15: Age 70 Withdrawal Report**

Jane's earned income lasts until she is age 65 and John is age 68. Social Security benefits begin when John turns 70 and double when Jane reaches age 70. As in the Base Scenario the After-Tax account is withdrawn over two years and in parallel with Tax-Deferred Account distributions.

## Conclusion

ORP brings the power of linear programming, widely used for commercial models, to the retail market via the Internet to present the big picture of saving for retirement and managing cash flow during retirement. ORP is not an accounting tool nor is it a risk evaluation tool. ORP offers guidelines and not precise values.

Mostly ORP is an educational tool meant to introduce the nascent retirement planner to the vocabulary and issues of the field. The documents listed in the reference give a broad and deep view of the retirement planning field.

ORP demonstrates that:

- The miracle of compounding, whereby yesterday's savings grow tomorrow, and more importantly tax-advantaged savings grow faster than conventional savings.
- Inflation is death for a retirement plan.
- Taxes matter; sensible and not necessarily obvious withdrawal techniques will increase the money available for retirement spending.
- Investment strategy greatly impacts the amount of money available for retirement.

The utility of ORP is its use to quantitatively evaluate retirement planning alternatives .

## Appendix A: Glossary

An *After-tax Account* is an account in which taxes are paid on all investments before making the investment, and taxes are paid on the returns as they are realized.

A *distribution* is the money taken out of the Tax-deferred Account to meet withdrawals, pay taxes or to be transferred to the After-tax Account.

An *early distribution penalty* of 10% is assessed by the IRS for distributions from tax-deferred savings that occur before the of age of 59 ½.

*Estate planning and investment strategies* are two topics that are outside the scope of this paper. They are represented in the ORP model by two parameters, set by the user but they are not implemented in the mathematics of the model.

*Life expectancy* is an individual matter but for purposes of modeling the IRS life expectancy tables is sufficient [11]

*Linear programming* [2] is a popular management science modeling technique used as a decision support tool in a number of different fields, ranging from running oil refineries, to locating warehouses, formulating chicken feed, and even extending into the financial world to perform portfolio optimization [3]. *Linear* means that the model is described as a system of linear equations. *Programming* means that the output of the model is a program to be followed to obtain the computed optimal result. There are several commercial, personal computer software systems available to solve linear programming models. All of these systems use a common expression of the problem. Once a problem formulated as linear model then any of these several system can solve it [26].

*Living expense* is the amount of money that the ORP computes, in today's dollars, which are available for spending in retirement.

*Math programming* is the field of constrained optimization that includes models with discrete and non-linear elements. Linear programming is the core of math programming.

The IRS requires a *minimum distribution* from tax-deferred savings either after the age of 70 ½ or retirement, whichever ever comes later. This rule does not apply to Roth IRAs.

*The model* is the mathematical description of the behavior, over time, of the Tax-deferred Account and the After-tax Account.

The *objective* of the linear programming model is to compute a program of disbursements, from the Tax-deferred and After-tax Accounts, that maximizes the money available to be spent during retirement or left in the retiree's estate. The program is

subject to the constraint of providing for inflation adjusted living expenses and the constraints imposed by the IRS.

The term *optimal* appears in title of this paper because the results described herein were obtained by formulating the retirement problem as an linear programming model. Values are assigned to policy parameters and the model is solved for the optimal retirement plan. The optimal solution shows the maximum objective that satisfies all constraints.

**Optimal Retirement Planning (ORP)** is a *linear programming* (LP) model [2], formulated using the commercial personal computer software Visual Math Programming [5], and solved by the BdLP optimizer [3]. The model is a system of linear constraints that ORP iteratively solves in such a way as the final solution maximizes the withdrawals. ORP consists of the VMP and BdLP runtime modules, a data preprocessor, a post-solution report generator, and an interactive user interface.

The *optimal retirement plan* is the schedule of account disbursements and withdrawals that maximizes the objective.

The *recalculation method* of computing the minimum distribution at age 70 ½ is as follows: Each year the minimum distribution is computed by determining a new life expectancy value from the IRS actuarial table and dividing that value into the Tax-deferred Account balance. Since you will always have a life expectancy larger than one, the Tax-deferred Account balance never goes to zero.

A **Roth IRA** is a tax-exempt, individual, retirement savings plan in which taxes have been paid on all contributions and all withdrawals are tax exempt. A Roth 401(k) is similar except it is administered by an employer on the same terms as a 401(k) plan.

A **Section 457Plan** is a deferred-compensation plan similar to a 401(k) except that it for government employees.

A **Tax-Advantaged** plan is a saving plan that offers advantages over conventional saving plans. A tax-advantage plan may be a tax-deferred or a tax-exempt plan.

A **Tax-deferred Account** is an account in which taxes have not been paid on either the contributions or the investment returns. Taxes are paid when money is distributed from the account. Individual Retirement Accounts (IRA), 401(k) plans, 403(b) plans and profit sharing plans (Keogh) are examples of Tax-deferred Accounts.

A **Tax-Exempt Plan** is a tax-advantaged plan whereby taxes have been paid on contributions to the plan but withdrawals are tax free.

The **Term Certain** method of computing the minimum tax deferred distribution amount is where the IRS provides a life expectancy figure for a person at the age of 70 and that value is reduced by one for each year thereafter. The minimum distribution is then the amount in the Tax-deferred Account divided by the life expectancy for a particular year.

The retirement plan's *value* is the total amount of money withdrawn from the plan plus the account balances at the end of the life expectancy, i.e. the *estate*.

A *withdrawal* is money taken out of the retirement accounts for living expenses, gifts, or left in the estate.

## Appendix B: IRAs and Roth IRAs

Analytically, there is little difference between a regular IRA and a Roth IRA, all other things being equal:

The distribution value of a *regular* IRA at year  $y$ , earning at rate  $r$ , starting with before-tax balance  $b$  and paying tax rate  $t$  is given by:

$$V = b \times (1 + r^y) \times (1 - t)$$

The value of a *Roth* IRA at year  $y$ , earning at rate  $r$ , starting with before-tax balance  $b$  and paying tax rate  $T$  at the time of the contribution.

$$R = b \times (1 + r^y) \times (1 - T)$$

There are 3 cases:

If  $t = T$  then  $V = R$  and whole thing is a wash. This case occurs when the distribution tax rate ( $t$ ) is the same as the pre retirement personal income tax rate ( $T$ ).

If  $t < T$  then  $V > R$  the Tax-deferred Account provides more retirement funds than does the Roth IRA. This case occurs when income tax rate during retirement ( $t$ ) is less than during the working years ( $T$ ). This is the normal retirement situation.

If  $t > T$  then  $V < R$  and the Roth IRA is a better deal. This case occurs when the distribution tax rate ( $t$ ) is larger than the tax rate before retirement ( $T$ ). This is the situation for very young wage earners, graduate students, and the like. Roth IRA savings during this period in a persons working life can have a big payoff.

No more can be said about this without getting into the area of financial planning.

## Appendix C: ORP Arithmetic

This appendix describes how ORP does its account arithmetic and the timing of account activities.

Table C is the first part of Table 4 earlier.

Age	Account Bal	Contrib	Distrib	MinDist
060	1400	10	0	0
061	1550	10	0	0
062	1715	11	0	0
063	1898	11	0	0
064	2098	11	0	0
065	2261	0	59	0
066	2339	0	147	0
067	2411	0	162	0
068	2507	0	145	0

**Table C: Tax-Deferred Account Activity Report**

1. The account balance is shown as of the end of the age year.
2. The account balance as of the beginning of the year is the account balance for the end of the previous year.
3. Withdrawals are made at the beginning of the year.
4. Contributions are made at the end of the year.
5. A year's investment return is made at the end of the year. It is computed as rate-of-return\*(beginning balance – withdrawal).

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## **About the Author**

James S. Welch, Jr. has been a professional computer programmer since he graduated from Antioch College in 1958. His particular interests are special purpose database management systems, large scale accounting applications using commercial database systems, and development of Mathematical Programming Systems, working in the areas of model specification, model reporting and high speed optimizers. As he approached the retirement years he decided to apply his Mathematical Programming skills to the question of strategic modeling of retirement; thus ORP.

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