Mitigating the Impact of Personal Income Taxes on Retirement Savings Distributions

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Abstract

When retirement savings include a large tax-deferred account distribution strategies for sequencing withdrawals from these accounts differ in the amount of money available for annual spending during retirement. The common practice for the scheduling of withdrawals from retirement savings accounts is to first deplete the after-tax account, then the tax-deferred and finally the Roth IRA. This paper quantitatively evaluates optimal plans that maximize spending by sequencing annual withdrawals to minimize the impact of taxes in order to achieve a targeted final total asset value. We show that the optimal retirement savings withdrawal strategy improves on common practice by increasing the money available for retirement spending by 3% to 30%. Most of the optimal withdrawal plans evaluated in this paper make withdrawals from the tax-deferred account across the entire span of retirement in parallel with withdrawals from first the after-tax account and then the Roth IRA later in retirement.

Keywords: retirement planning, withdrawal strategy, Roth IRA, tax-deferred savings, linear programming, optimal distribution plan, retirement spending

Mitigating the Impact of Personal Income Taxes on Retirement Savings Distributions

Many retirees have multiple types of retirement savings accounts with different distribution characteristics[1]. These retirees seek a strategy for the annual withdrawal of funds from their savings in a way that will maximize **spending[2]**, not exhaust savings prematurely, and not leave a large surplus.

The answer to a problem depends on how we phrase the question. In retirement planning the question is usually along the lines of "Given my living expenses when will my savings run out?" The answer comes in the form of which year the savings are exhausted. This idea is extended to the Monte Carlo method which computes the probability of plan failure due to asset volatility.

Professor William F. Sharpe [2013] succinctly defines the problem in his blog:

> *It seems to me that first principles dictate that any rule for spending out of a retirement account should at the very least adhere to the following principle:*
>
> *The amount you spend should depend on*
>
> 1. *How much money you have, and*
> 2. *How long you are likely to need it*

In other words, given my assets, my estimated life expectancy and my desire to spend all my savings, leaving some predefined **final total account balance (FTAB)** what is the maximum amount of money that I will have to spend each year, after taxes?

This paper addresses retirement planning from the perspective of maximizing spending.

[1] Appendix A is a glossary providing the narrow definition of terms used in this paper.
[2] Spending is the money available for annual personal consumption.

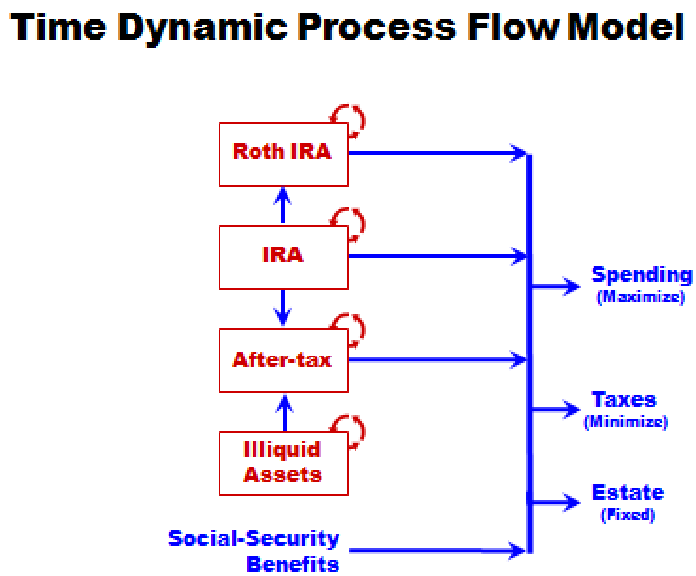

Figure 1: Overview of Retirement Planning

Figure 1 presents an overview of the retirement financial model. The **process** is the storing and distribution of funds over the term of retirement (**time dynamic**) [Hirshfeld 1969]. The boxes inventory money which increases in value each year (circular arrows) according to a **rate of return (ROR)**. The top three boxes are liquid savings accounts from which any amount of money can be withdrawn for spending, taxes or to fund the estate (FTAB).

- Money flows from the Roth IRA to spending or the estate.
- Money flows from the **tax-deferred account (IRA)**[3] to the **Roth IRA** or the **after-tax account**, as well as spending, taxes or the estate.
- Money flows from the after-tax account to spending or the estate.
- Illiquid assets can only be sold as a complete entity and the sale proceeds, after taxes are deducted, are transferred to the after-tax account to be distributed later.
- Social Security benefits and pensions are other sources of funds that are not part of savings but contribute to spending, taxes, or the estate.

[3] We use the term IRA to represent all tax-deferred accounts. (See Appendix A)

From the perspective of how they are taxed, these accounts are four entirely different entities.

We omit Social Security benefits and Illiquid Assets from this study because we want to concentrate on the impact of personal income taxes generated by tax-deferred account distributions on spending.

Distribution strategies for sequencing withdrawals from retirement savings accounts differ in the amount of money they produce for spending during retirement because of income taxes on tax-deferred account distributions.

There are two issues in devising a strategy for making annual withdrawals:

1. The order in which accounts are selected for withdrawal which affects the amount of money available for spending.
2. The amount to be withdrawn each year which affects the FTAB.

Issue 1 – Order of Withdrawal

The order of annual withdrawal affects spending because spending is reduced by personal income taxes paid on tax-deferred account withdrawals. Under the Federal progressive income tax a large IRA distribution is taxed at a higher rate than several small distributions made in different years. There are no taxes on distributions from the other two accounts.

The **common practice** is to withdraw savings from retirement accounts in this order:

1. The after-tax account until it is depleted,
2. The IRA until it is depleted, and
3. The Roth IRA account to the end of the plan.

Issue 2 – Amount of Withdrawal

The size of annual distributions determines the plan's final deficit or surplus. Too large of an annual distribution will cause savings to be exhausted before the end of the plan; a great concern to many retirees. A surplus is not of such great concern but an excessive surplus means that the retiree may have practiced unnecessary self-denial while seeking insurance from the deficit threat.

The common practice is to estimate retirement spending through analysis of pre-retirement spending and retirement budget planning. Using this estimate a retirement calculator may be used to determine the plan's deficit or surplus. A variation of this approach is to utilize a

Monte Carlo method calculator to estimate the probability of plan failure, i.e. running out of money before the plan end.

A Unified Approach

We compare results from two computer programs that have a unified approach to these two issues:

1. A **simulator** that implements the common practice for Issue 1 (fixed order of withdrawal) and computes maximum savings withdrawal amounts and thus maximum spending (Issue 2).
2. A **linear programming optimizer** that maximizes spending by computing both the optimal account withdrawal order and the withdrawal amount.

Both programs report a schedule of annual withdrawals; the **withdrawal plan**.

The fundamental difference between these two programs is that the order of account withdrawal is defined in the simulator whereas the optimizer computes the optimal account withdrawal sequence.

Discussion

Regarding optimization, this paper describes how linear programming is used to maximize spending throughout retirement while leaving a specified balance in retirement savings; i.e.no plan failure and no large surplus. The result is an efficient retirement savings withdrawal plan that provides a steady stream of inflation adjusted money over the term of retirement.

Linear Programming (LP) is an operations research tool that has been a successful computer application since the 1950's [Orchard-Hays 1984]. An LP model is solved by commercial computer software that accepts a model consisting of a set of activities that can be done within constraints on those activities. From the large universe of model solutions the LP optimizer computes one that has some maximum economic value. The objective of an LP model is to optimize an economic value that, in this paper, is the retirement planning value of spending. LP mathematically guarantees that there is no better solution than the one computed [Danzig 1963]. The optimizer reports the economic value of the model and the activities that contribute to the solution.

This paper demonstrates scenarios in which linear programming is used to compute withdrawal plans that increase spending in the range of 3% to 30% as compared to common practice.

A common criticism of simulators and optimizers is that they do not reflect the market volatility of asset values and returns. We argue that active, capital preservation portfolio management can dampen the adverse effects of market volatility so that, when combined with a long planning horizon of 25 years or more the fixed return assumption is valid for planning purposes. Given this assumption, we compare the efficiency of two methods of retirement income planning while ignoring the question of longevity (defined as the chance of exhausting savings before the end of the planning horizon), as is commonly measured with the Monte Carlo method. We assume market volatility and associated risks would impact both methods similarly.

In this paper we seek to establish the credibility of the two computer programs, measure the differences between the two approaches for various scenarios, and discuss the dynamics of their resultant withdrawal plans.

Literature Review

The common practice is based on quantitative studies.

Raabe and Toolson [2002] showed that the common practice of withdrawing from retirement savings is more efficient than any other permutation of sequential account distribution strategies. Their approach recognized the interaction between the IRA and the after-tax account.

Saftner and Fink [2004] compared the results of retirement savings that are exclusively in one of the three accounts. Their results showed that saving in a Roth IRA and an IRA will result in the same plan value (spending plus the FTAB). They showed that the after-tax account is less efficient than the other two accounts because of the reduced compounding that results from after-tax account income being taxed as it is incurred. When employer contributions are added to employee saving, the tax-deferred account is superior to the other two.

Horan [2006] studied the question of whether to withdraw from the IRA or the Roth IRA first. He compared two “naïve” models, 1) Withdraw from the IRA first and the Roth IRA second; 2) vice versa, to his “informed” method. He concluded that withdrawing from both accounts in parallel is the most efficient strategy. He distributed from the IRA until it reached the top of the current tax bracket and then satisfied any remaining spending requirements from the Roth IRA. Personal income taxes were modeled.

We know of two published papers describing LP models that compute optimal retirement savings withdrawal plans.

Ragsdale, Seila and Little [1993] demonstrated that their LP optimal withdrawal plan is superior to two heuristic withdrawal methods. Withdrawals are made from two tax-deferred accounts with differing rates of return. Their model fixed the withdrawal rate and maximized generated plan surplus. They computed personal income taxes on withdrawals, met the **Required Minimum Distribution (RMD)**, minimized the Excess Distribution Penalty (no longer a feature in the tax code), and minimized estate taxes. They modeled two IRAs with different RORs and concluded that distributing the lower performing account first is optimal.

Coopersmith and Sumutka [2011] compared the results of their **Tax Efficient (TE)** linear programming model to their common rule. Their model computed personal income taxes on tax-deferred withdrawals plus Social Security benefits, satisfied the **RMD** and minimized estate taxes. TE showed improvement over common practice for situations where

- The after-tax account ROR is greater than the tax-deferred ROR.
- Initial after-tax account savings are greater than 10 percent of total retirement savings.
- Itemized deductions are greater than the standard deduction.

We extend this prior work by:

- Holding the FTAB constant and maximizing spending,
- Modeling all three accounts and their interaction,
- Eliminating TE's restrictions,
- Implementing IRA to Roth IRA conversions.
- Assigning a single ROR to all accounts to concentrate on the effects of taxes on the retirement plan.

The Experiment

Our experiment is to compare the common practice retirement plans to optimized plans. There are three elements of the experiment; the modeling software, the situation being modeled, and the scenarios for obtaining the computational results.

The Software

We used two computer programs:

1. The **Common Practice Simulator (CPS)** is an Excel spreadsheet that we use to simulate the common practice for scheduling account withdrawals and compute maximum withdrawals.
2. The **Optimal Retirement Planner (ORP)** is the linear programming system that we used to compute the optimal plans for this study.[4]

CPS is based on a generally recognized heuristic but with its direction reversed, i.e. set the FTAB to zero and maximize spending. ORP maximizes spending for a zero FTAB in a manner that directly compares to CPS.

The two programs use the same parameter set and compute to the same objective: maximum spending. The computed plan is measured by spending at age 66 in today's dollars. Spending for subsequent years is this amount adjusted for inflation; i.e. the anuitization of spending. Both programs model the Federal progressive income tax and the RMD using 2014 IRS tables.

Given a set of parameters both programs' objective is to compute the maximum spending level that will leave a zero balance in the FTAB.

The Situation

The situation being modeled is a single, 66 year old retiree with $1,000,000 in retirement savings and a planning horizon of 29 years (to age 95). The FTAB is zero, i.e. there is no estate.

All three retirement savings accounts assume the same ROR. The purpose of this study is to demonstrate the impact of personal income taxes on the optimal withdrawal plan without the need to address the confounding impact of different RORs for the three accounts. [Coppersmith and Sumutka 2011].

Annual inflation is assumed to be 2.5%.

[4] The CPS spreadsheet is available on request. ORP may be found at www.i-orp.com.

Computational Results

The experiment was to run the two programs with the parameter set described above, for different scenarios, and compare their results.

Rates of Return

In the ROR scenarios one million dollars of retirement savings are distributed across all three accounts. The IRA contains $400,000, the Roth IRA $350,000 and the after-tax account $250,000. These proportions were chosen by computing accumulation phase savings for a 30 year old who allocates 1/3 of her annual retirement savings to each of the three accounts. The accumulated asset totals were evaluated at age 66. The Roth IRA account balance is lower than the IRA because of income taxes deducted from the Roth IRA contributions. The initial after-tax account balance is even lower due to income taxes deducted from contributions and because the 15% capital gains tax paid on annual investment returns reduces compounding [Saftner and Fink, 2004].

The scenarios compare CPS and ORP spending for a range of RORs. Recall that for the purpose of comparison, RORs are considered average rates and the volatility of the RORs would impact both methods similarly.

ROR selection is one of the important discretionary choices that the retiree has to make. A low ROR indicates a willingness to sacrifice return to achieve low portfolio volatility. A high ROR indicates a desire to achieve greater return by tolerating a higher level of volatility. Since these models are deterministic, not probabilistic, their results are more realistic for low RORs.

ROR scenario summary. Table 1 compares spending in today's dollars, for plans computed by CPS and ORP, using a range of RORs.

Table 1: Comparison of CPS to ORP

ROR %	Spending - $000		Efficiency %
	CPS	ORP	
0.5	22	25	13.6
1	24	27	12.5
2	28	31	10.7
3	33	35	6.1
4	38	40	5.3
5	43	45	4.7
6	49	51	4.1

ROR %	Spending - $000		Efficiency %
	CPS	ORP	
7	54	56	3.7
8	60	62	3.3
9	66	68	3.0
10	72	75	4.2
15	103	107	3.9

Column **ROR %** contains the rate of return parameter that was varied for the results in Table 1.

The **Spending** columns show each program's maximum spending. A year's withdrawals can come from of any combination of accounts, their sum minus taxes will equal spending for that year.

The **Efficiency** column quantifies the advantage of the ORP withdrawal plan over CPS. We define efficiency to be the spending difference as a percentage of CPS spending at age 66:

Efficiency = (ORP spending – CPS spending)/CPS spending

The Efficiency column indicates that as the RORs grow the advantage of optimization over common practice diminishes. As discussed earlier, small RORs indicate less volatile assets and the constant ROR assumption is more credible. Thus ORP spending improvement is more relevant to conservatively invested accounts. ORP's improved spending partially compensates for reduced returns on lower risk savings.

The 5% ROR plan. This section explores how ORP and CPS determined the spending levels for the 5% ROR scenario.

Withdrawal plan. Withdrawal scheduling is selecting one or more accounts and determining the amount to withdraw each year. CPS account selection is defined as part of the algorithm. ORP computes the account and the amount for each year's withdrawal. Figure 2 shows the distribution plans reported by CPS and ORP.

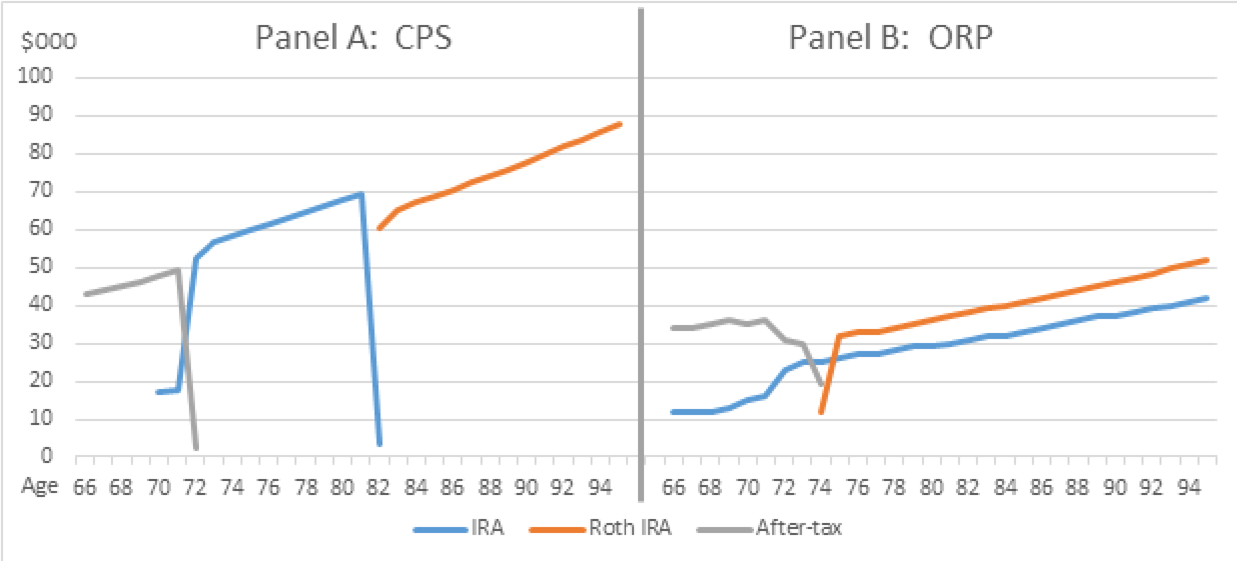

Figure 2: Annual Distributions by Account, 5% ROR Scenario

Panel A of Figure 2 shows CPS withdrawals for each year of the 5% ROR scenario. The after-tax account distributes until age 70, when the RMD forces the first IRA distribution. The IRA makes distributions until it is depleted at age 82 when the Roth IRA takes over. IRA withdrawals overlap withdrawals from the other accounts only at the boundaries. The IRA withdrawals are elevated above the other two lines because of extra money withdrawn to pay taxes.

Panel B is the ORP withdrawal plan. The after-tax account and the IRA make parallel distributions until age 70 when the RMD begins. Then IRA distributions are just large enough to push taxable income to the top of the 10% bracket (See Figure 4). The RMD increases the IRA distributions and reduces the after-tax distributions. After the after-tax account is depleted the IRA and Roth IRA make parallel distributions. IRA distributions are maintained at a level sufficient to hold taxable income at the top of the 10% tax bracket while the Roth IRA satisfies remaining spending requirements

Savings account balances. Figure 3 shows the behavior of the account balances over time under the distribution plans shown in Figure 2.

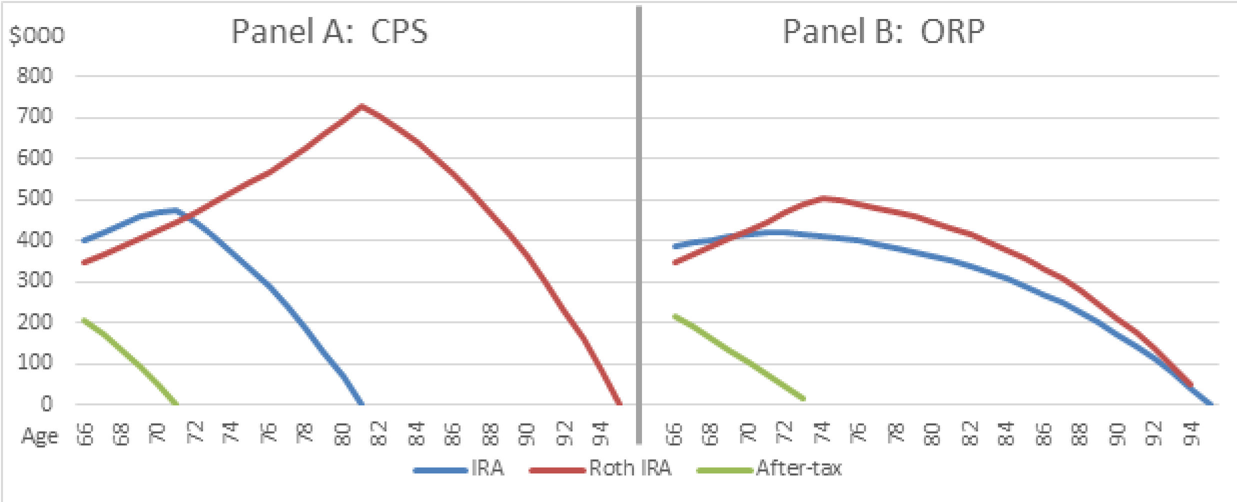

Figure 3: Savings Account Balances, 5% ROR Scenario

In Panel A the annual asset balances are falling in tandem as expected for the common practice. In both panels the IRA and Roth IRA continue to accumulate as the after-tax account declines.

In Panel B after ORP depletes the after-tax account the IRA and the Roth IRA decline in parallel.

Income taxes. Figure 4 shows how nominal (de-inflated) IRA distributions are allocated across the Federal income tax brackets.[5]

[5] Both models were run with 2.5% inflation then the tax reports were "de-inflated".

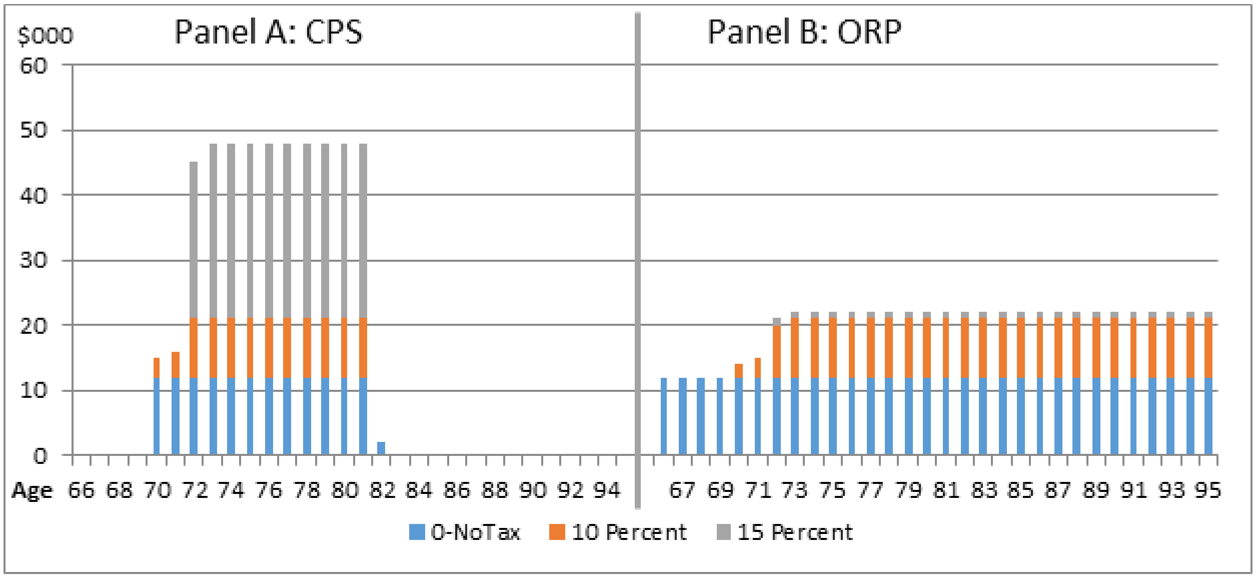

Figure 4: Nominal Income Tax Brackets, 5% ROR

Each vertical bar represents income subject to taxes. Each bar is segmented into parts according to the tax bracket that the income falls into. For example, in Panel A, the age 72 bar shows income divided into the No Tax, 10% and 15% brackets. The *No Tax* bracket includes the standard deduction, one personal exemption, and the allowance for being over 65..

Panel A shows the CPS income tax brackets. When there are no IRA distributions there are no taxes. During IRA distributions, CPS taxable income climbs into the 15% tax bracket. (See Figure 2).

Panel B shows ORP income tax brackets. After the after-tax account is depleted IRA distributions fill up the 10% bracket (See Figure 2). This process is being driven by the annualitization of spending and the zero FTAB requirement.

CPS is paying all of its taxes early in the plan while ORP spreads taxes across the plan at an overall lower level.

Table 2 shows taxes paid in both real (Inflated) dollars and nominal (de-inflated) dollars for the 5% ROR scenario.

Table 2: Total Taxes Paid, 5% ROR

System	Taxes - $000	
	Real	Nominal
ORP	33	21
CPS	64	50
Difference	31	29

The difference between the program's taxes are not consistent with ORP's $2,000 spending advantage reported in Table 1. Compared to each other the differences are dramatic. But $29,000 spread over a retirement of 29 years leaves $1,000 of spending unaccounted for. We conjecture that the timing of tax payments is as important as the magnitude of tax differences. CPS pays more taxes early in the plan which reduces IRA compounding and thus reduces spending.

Account allocation. An interesting question is "*How much do these results depend on the initial allocation of funds in the savings accounts?*"

Table 3 summarizes the difference between CPS and ORP for a selection of starting account allocations using a 5% ROR. The first column shows the percentage of the one million in savings allocated to each account. The first row is the 5% ROR scenario from Table 1.

Table 3: A Sampling of Initial Account Balances, 5% ROR

Allocation IRA/ROTH/AT	Spending - $000		Efficiency %	Discussion
	CPS	ORP		
40/35/25	43	45	4.7	From Table 1
00/50/50	45	45	0.0	Low IRA initial balance provides low levels of taxes to work with.
30/50/20	45	46	2.2	
50/50/00	44	45	2.3	Parallel IRA and Roth IRA distributions at top of 15% bracket until IRA is depleted at age 86.
50/30/20	44	45	2.3	Large IRA , Roth IRA, and after-tax balances
30/00/70	33	43	30.3	No Roth IRA. From age 70 (RMD start) ORP distributes from after-tax and IRA in parallel.
40/00/60	33	43	30.3	
50/00/50	34	43	26.5	
60/00/40	36	43	19.4	
70/00/30	38	43	13.2	
80/10/10	43	43	0.0	High IRA balances with low after-tax balances means that small parallel distributions have little
90/10/00	42	42	0.0	

Allocation IRA/ROTH/AT	Spending - $000		Efficiency %	Discussion
	CPS	ORP		
90/00/10	42	42	0.0	effect.
100/0/00	42	42	0.0	

The five scenarios with no Roth IRA balance show efficiencies that are significantly larger than the rest of the results. ORP takes full advantage of the strategy of distributing the IRA and after-tax accounts in parallel to the end of the plan as in the 40/00/60 scenario or depleting the after-tax account near the end of the plan as in the 60/00/40 scenario. When there is a Roth IRA balance present then ORP distributes the IRA in parallel with the other two accounts but depleting the after-tax account before beginning Roth IRA distributions, similar to the common practice.

It is well to remember at this point that each optimal solution is the best available for the given circumstances.

Account Size

Table 4 summarizes spending and efficiency for retirement accounts of different sizes using the 5% scenario. The first column shows the dollar amount of the total portfolio, distributed across the three accounts in the same proportions as the ROR scenarios.

Table 4: Different Initial Account Balances, 5% ROR

Beginning Balance	Spending - $000		Efficiency %
	CPS	ORP	
1 Million	43	45	4.7
2 Million	83	89	7.2
3 Million	123	131	6.5
4 Million	163	172	5.5
5 Million	201	214	6.5

ORP's efficiency is not sensitive to the amount of retirement savings.

IRA to Roth IRA Conversions

All of the results reported thus far were produced with no **IRA to Roth IRA conversions (Roth conversions)**. The experiments were repeated with conversions allowed.

For every pair of scenarios the total amount of real spending increased by less than $1,000. The withdrawal plans differed but the end results were the same.

For example, both of the two 5% ROR, IRA only, scenarios (bottom row of Table 3) compute a spending level of $60,000 and a total plan value of $2,629,000. Any improvement due to Roth conversions was lost in rounding error. Hardly worth the extra paper work!

An exception was that the zero Roth IRA scenarios in Table 3 showed a $1,000 spending increase when conversions were allowed.

Figure 5 compares nominal IRA distributions for the two 5% ROR scenarios, one with no Roth conversions, the other with conversions allowed. Both scenarios have the same amount of nominal income subject to taxes every year. Conversions move the tax payments from the end of the plan to the front.

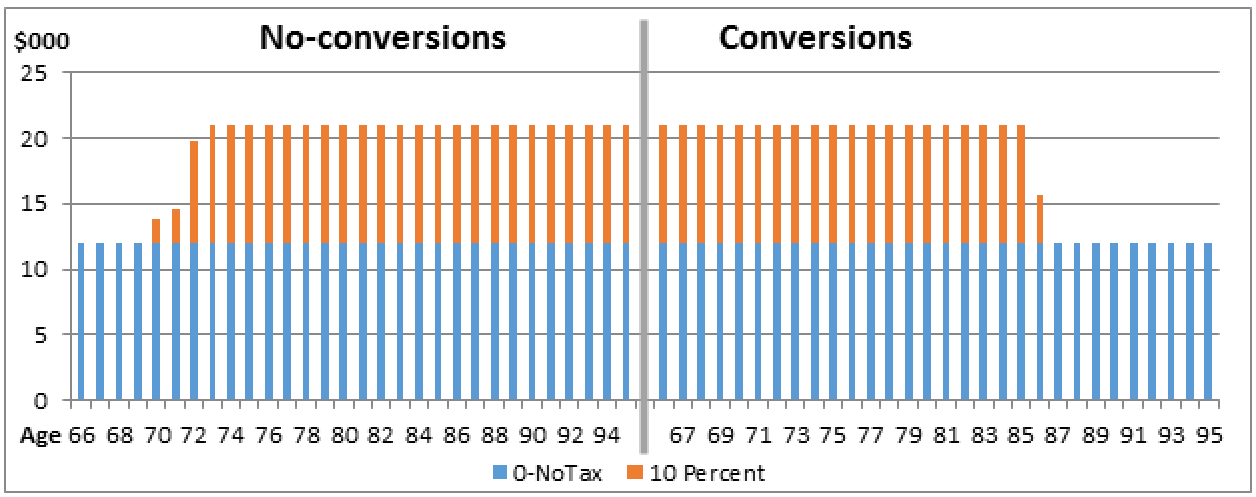

Figure 5: Nominal Tax Brackets for IRA to Roth IRA Conversion Scenarios, 5% ROR

Early in the no-conversion scenario IRA distributions are at the top of the no-tax bracket. The after-tax account supplements the IRA to meet spending requirements.

IRA withdrawals for Roth conversions are at the top of the 10% bracket while the after-tax account contributes to spending. Late in the plan IRA distributions drop back to the top of the 10% bracket as the Roth IRA supplements spending.

Less total nominal tax was paid ($21,000 over the entire plan) when conversions were permitted then when not ($33,000) even though annual spending was the same. Similar to the Figure 4 discussion we conclude that this is because the reduction in the IRA balance early in retirement meant lower IRA compounding of returns throughout retirement. The compounding loss was offset by the reduction in taxes paid.

Roth conversions may be preferred when factors other than economics are taken into account; e.g. the anticipation of an increase in Federal income tax rates or the desire to leave a substantial estate in a tax free account. Also, the retiree must assess the effect of income on Medicare premiums, given that Roth conversions affect annual incomes in each of the income dependent Medicare premium categories.

Model Validation

Before accepting these results we need some assurance that they are valid. We validate our programs by comparing CPS and ORP results for degenerate scenarios and comparing ORP results to other, independent models of similar purpose.

Degenerate Scenarios

Our degenerate scenario tests are based on a model with the full $1,000,000 in only one account and with nothing in the other two accounts. Since there is only one active account there is no interaction between accounts and the spending values computed by CPS and ORP should be the same.

Table 5 compares CPS and ORP spending for the degenerate scenarios.

Table 5: Degenerate Scenario Comparisons, 5% ROR

Scenarios	Spending - $000	
	CPS	ORP
After-tax	42	42
Roth IRA	46	46
IRA	42	42

Considering how different CPS and ORP are from each other (Excel vs FORTRAN) the computed values for the degenerate scenarios are acceptable.

For the IRA-only scenarios both systems compute personal income tax on withdrawals. The results indicate that the two programs' income tax calculations are consistent.

Compare ORP to Other Models

The second test is to compare ORP's results to that of other retirement calculators. This is generally not practical because conventional retirement calculators do not include progressive income taxes in their computations. We are aware of four published papers with computational results that can be meaningfully compared to ORP.

Table 6 compares ORP results to the results of the four other models. These models are all of the *specify-spending-and-compute-the-FTAB variety*. We compare ORP to these models by having ORP assume their computed FTAB along with the term of their plan, and compute the optimal spending levels to see if ORP's ending results compare to the models' beginning values.

Table 6: Compare ORP's Spending to Those of Other Models

Model	Term Years	FTAB $000	Spending - $000	
			Model	ORP
CTM	30	1,608	80	78
TE	25	824	60	62
Reichenstein [2006]	30	0	103	105
Reichenstein [2013]	33	0	37	34

CTM is the Comprehensive Tax Model by Sumutka, et al [2012]. CTM assumes a spending rate and computes the savings account balance at the end of the plan.

TE is the "Tax-Efficient Retirement Withdrawal Planning model by Coppersmith and Sumutka [2011]. TE assumes a spending rate and maximizes the savings balance at the end of the plan.

Reichenstein assumes a zero FTAB and computes the age at which all savings are depleted.

Of course, we could argue that that our comparison validates these models.

Conclusion

We have demonstrated that:

- Linear programming is a credible retirement planning tool.
- Common practice, as represented by CPS, is an efficient but suboptimal withdrawal strategy.
- Minimizing taxes is only part of the optimal schedule. When higher taxes are paid early in the plan then spending is reduced by reduced account compounding.
- Optimization is the balancing of asset compounding against minimizing taxes.
- Optimization improves on common practice, as represented by CPS, by 3% to 30%.
- If there is no IRA then there are no income taxes and optimization follows common practice.
- Optimization shows a significant advantage over the common practice for scenarios with similar IRA and after-tax account balances and no Roth IRA.
- Partial IRA to Roth IRA conversion decisions may be based on considerations other than plan economics.

The practical consequences are that for a retiree with multiple retirement savings accounts, there are important decisions to be made about funding next year's spending. The choices include:

1. Which account(s) to withdraw from?
2. How much to withdraw?
3. Make a partial IRA to Roth IRA conversion?

The decisions hinge on the year's projected income, subject to anticipated personal income taxes, in the context of the overall retirement picture. Then, she has to revisit the process each year with new data that reflect her changed circumstances, i.e. spending increases/decreases.

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Appendix A: Glossary

After-tax Retirement Savings Account: Contributions to the after-tax account can be from any source that has been taxed. Taxes are paid annually on asset sales' profits, dividends and interest. Withdrawals are not taxed. When the IRA withdrawal exceeds spending, say, due to the RMD, the surplus is transferred from the IRA to the After-tax account. Taxes, at the capital gains rate, are assumed to be paid annually, thereby reducing the account's ROR. This reduced after-tax ROR is the main reason that both common practice and ORP distributes the after-tax account first.

After-tax accounts typically include common stock, which often pay dividends that are subject to income tax. We assume that the after-tax account is invested only growth in stocks or mutual funds which pay insignificant dividends relative to the rest of the plan. Also, since the after-tax account is drawn down first there are no dividends later in the plan.

The literature frequently uses the term taxable account for what we call the after-tax account. In our view all accounts are taxable because they are taxed either as money enters the accounts or as it is distributed.

Efficiency: the percentage improvement of one model's results over another.

Estate: The plan's FTAB**.** This is a non-negative number specified as part of a scenario's assumptions.

Final Total Account Balance (FTAB): The sum of all three savings account at the end of the plan. FTAB is also known as the plan's estate. The FTAB is a settable parameter. It is set to zero for the scenarios in this paper. Positive values are required for comparing ORP to other systems.

Heuristics: business rules used to recommend a decision. The heuristic of interest here is the common practice for sequencing of accounts for retirement savings withdrawal.

Illiquid Asset: An asset that can only be sold as a single entity. A home, business or partnership are examples of illiquid assets. The proceeds of the sale less capital gains taxes on any profits are transferred to the after-tax account in the year of the asset sale.

Optimization finds the "best available" value of some objective function given a defined domain. For retirement savings distribution the domain is the retiree's financial situation and choices to be made. The objective function (economic value) is the amount of money available for spending. Optimization is the balancing of asset compounding against minimizing taxes.

Rate of Return (ROR): The profit on an investment expressed as a percentage of investment's value.

Required Minimum Distribution (RMD): The RMD is an amount that the IRS requires be withdrawn from the IRA annually beginning at the age of 70½. It is computed as the IRA account balance on December 31 of the previous year divided by a life expectancy value taken from an IRS published table [IRS 2014]. The RMD is recomputed annually with a different life expectancy divisor. RMD withdrawals cannot be converted to the Roth IRA.

Roth conversion: the partial distribution of funds from the IRA to the Roth IRA after personal income taxes have been paid.

Roth IRA Retirement Saving Account: Income taxes are paid on the employment income contributions to a Roth IRA but there are no taxes on withdrawals. In addition to employment contributions withdrawals from an IRA may be converted to a Roth IRA after personal income taxes have been paid on the withdrawals. After age 59 ½ withdrawals can be made from the Roth IRA in any amount without penalty.

Simulator: imitates a heuristic's behavior over time.

Spending: the amount of money, after taxes, available for retiree consumption expressed in today's dollars. Spending is money that leaves the model. ORP balances tax minimization against maximization of asset returns to maximize spending.

Tax-deferred Retirement Savings Accounts (IRA): There are no income taxes on employment earnings contributed to the IRA but all withdrawals are taxed as personal income. This type of account includes IRA, 401k, 403b and a variety of others, all of which are generically equivalent for purposes of this study. The term IRA is used to denote the collection of these accounts since most are converted into an IRA before or at retirement. After age 59 ½ withdrawals can be made from the IRA in any amount without penalty.

Withdrawal Plan: The amount of money withdrawn from each account each year. The plan includes money used for taxes, Roth conversions, spending, and the FTAB.

Appendix B: ORP Calculations

Modeling progressive income taxes in an LP model is straightforward.

ORP maximizes spending. Taxes reduce spending. LP increases spending by reducing taxes. Income subject to income taxes includes all IRA distributions without regard to their use, i.e. spending, Roth Conversions, or transfers to the after-tax account.

The optimal solution will assign income to the zero tax bracket first because it does not affect spending. Taxable income (income beyond the exemptions and deductions) goes into the 10% bracket, the bracket with the next smallest effect on spending. And so on up through the brackets. All brackets, except the 39.6% bracket, have an upper limit on how much money can be assigned to them. We use the 2014 tax tables in all income tax calculations.

ORP and CPS use the same account arithmetic:

1. Account balances are reported as of the end of the age year after the year's distribution and compounding.
2. The account balance as of the beginning of the year is the account balance at the end of the previous year.
3. Withdrawals are made at the end of the year after the account has been credited with its annual returns.
4. Contributions and Roth conversions are made at the end of the year.
5. A year's investment return is at the end of the year. The year's ending balance is computed as:

   (1+ROR)* beginning balance – distribution.

   This is the previous year's ending balance increased by the account's ROR prior to making distributions.
6. Taxes are paid for with account withdrawals and not Social Security payments.

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