A Three-Step Procedure for Computing Sustainable Retirement Savings Withdrawals

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Executive Summary

- Retirement funding has evolved from employer-managed pensions to retiree self-managed, tax advantaged savings.
- Part of retirement planning is to coordinate income from multiple sources such that savings are not depleted prematurely, no large undesired surplus is left behind, and disposable income is maximized.
- This paper describes 3-PEAT, a practical, retirement savings withdrawal procedure that addresses these objectives.
- The heart of 3-PEAT is a linear programming application that computes an optimal retirement income management plan whose results include necessary savings withdrawals and maximum disposable income for the term of the plan.
- Only the first year’s results are implemented; withdraw 3-PEAT’s recommended withdrawal amount from retirement savings and budget the years spending to 3-PEAT’s computed disposable income.
- Next year an entirely new plan is computed from revised initial conditions and its first-year results are implemented.
- 3-PEAT requires that the retiree adapt to variable annual disposable income. 3-PEAT increases withdrawals in rising markets and reduces withdrawals in down markets.
- Since 3-PEAT is a procedure, it is amenable to simulation.
This paper reports 3-PEAT simulations for six historical stock market periods. The results show that when 3-PEAT is compared to conventional variable withdrawal methods, it increases disposable income, prevents savings from being depleted prematurely, and reduces the variability of annual disposable income.

**Introduction**

Postings on some retirement forums indicate that some participants are using the Internet based **Optimal Retirement Planner (ORP)** to self-manage their retirement savings withdrawals. These users enter their parameters and compute a retirement plan that includes the optimal savings withdrawal for the current year. A year later they come back, complain about changes to the user interface, and compute a new plan with a new optimal withdrawal. Their input revisions reflect their changed circumstances; they are a year older, a year closer to the end of the plan, their Social Security income has increased, and their savings value has changed. Each annual iteration prepares a new plan for the remaining retirement. Users make their year’s savings withdrawal decision from the first year of the new plan’s report. The withdrawal covers disposable income and income taxes. They budget their year’s spending to the plan’s first year disposable income. The withdrawal and disposable income amounts are computed in the context of a full, optimal retirement income management plan.

**3-PEAT** formalizes what these users are doing on an ad hoc basis.

3-PEAT uses linear programming (a rigorous optimization method) for the crucial step of computing the optimal annual savings withdrawal. The optimal plan maximizes disposable income by scheduling savings withdrawals in a manner that balances the minimization of taxes against the maximization of asset returns.
3-PEAT will not fully deplete saving prematurely nor will it end with an unwanted surplus because it treats each plan’s **Final Total Asset Balance (FTAB)** as a constraint. 3-PEAT varies annual disposable income according to the value of retirement savings, plan length, and inflation. 3-PEAT confines retiree spending to within the limits of varying levels of disposable income thereby avoiding prematurely depleting retirement savings.

Since 3-PEAT is a **procedure**, it is amenable to simulation. The **Variable Withdrawal Simulator (VWS)** simulates the actions of a retiree using 3-PEAT to execute a 30 year retirement plan. This paper reports the results of experiments performed using VWS.

Bengen’s (1994) idea of back testing over historic stock market periods was adopted using Shiller’s (2016) S&P 500 index database. VWS’s essence is to operate 3-PEAT as though the present day retirement framework of tax-deferred accounts, Social Security income indexed to inflation, progressive taxation, index funds, etc. existed in historic market situations where most of these features had yet to be implemented. The chosen historic periods were:

1. **Modern Era**: 1985-2015, included the dot.com bubble burst and the Great Recession.
2. **Bengen’s Big Bang**: 1973-2003, began with the 1973 sell off. Guyton and Klinger (2006) named this same period the “**Perfect Storm**” because “… it included a severe market decline in the initial retirement years combined with inflation in the first decade of retirement that was three times the long term historical norm.”
3. **Stagflation**: 1966-1996, a period of mediocre market performance, high inflation (by U.S. standards), and a stagnant economy.
4. **Interbellum**: 1915-1945, the period where two World Wars bookended the Great Depression.
5. **Worst Case:** 1903-1933 was one of only two 30 year retirement periods in which the S&P 500 index finished lower (7.09) than it began (8.46) for a slope of -0.05. (The other, which began three years before Interbellum, had a slope of -.01.)

6. **Gilded Age:** 1880-1910, featured the panics of 1901 and 1907.

3-PEAT does not allow plan failure. **Plan failure** occurs when the amount withdrawn from retirement savings exceeds the saving balance. 3-PEAT computes withdrawals based on optimally scheduling savings withdrawals across the remainder of retirement in a tax efficient way.

The annual retirement plans computed for each era are optimal, i.e. there was no mathematically superior solution to the annual plans (Dantzig 1963). The 30 year plan was feasible in that it did not fail, it did not leave a large, unplanned surplus at the end, and it minimizes the amount of downside variability in its computed disposable incomes.

The goal of the study is to determine if the retirement forum contributors are practicing a safe retirement practice. The finding is: **3-PEAT is safe, effective, practical, and profitable.**

Following the literature review 3-PEAT and VWS are defined and computational results are reported. VWS was used to back test 3-PEAT against the S&P 500 index for six historical periods and compared it to three conventional variable withdrawal methods. Appendix A demonstrates how VWS simulates a retiree using 3-PEAT to manage her retirement income for the thirty years of a full term retirement.

**Literature Review**

Retirees want both the stability of constant spending and the rewards of investing in equity markets (Finke and Blanchett 2016). Bengen (1994) was the first to address this issue with his famous **4% Rule.** He reported that fixing the annual withdrawal amount at 4% of initial savings, indexed to inflation, would
fund a 30 year retirement. Current researchers seek methods that establish higher percentages of retirement savings that can be spent without increasing the probability of plan failure.

Guyton and Klinger (2006) introduced limited withdrawal variability into their inflation indexed, constant spending model. They began retirement with a heuristically determined initial withdrawal rate of some small percentage (e.g. 4%) of the initial retirement savings balance. From this they compute the initial withdrawal amount. Each year the year’s withdrawal amount was the initial withdrawal amount indexed to compounded inflation. They defined the year’s withdrawal rate to be the year’s withdrawal amount divided by the year’s savings balance. They then applied a set of decision rules to “enhance” the withdrawal rate so that plan failure was to be avoided. Their decision rules were based on the relative values of the initial withdrawal rate and the year’s withdrawal rate along with market returns and changes to the consumer price index. For some years this adjusted the year’s withdrawal rate which, in turn, adjusted the year’s withdrawal amount.

The difficulty with constant withdrawal rate methods, as noted by Guyton and Klinger, is that throughout retirement withdrawal amounts are tied to the initial withdrawal rate/amount which may have been unsuitable for a “perfect storm” economic environment that developed shortly after the initial withdrawal. This is the sequence of returns risk (Kitces 2014) when a reasonably constructed initial withdrawal amount turns out to be inappropriate for a difficult economic environment that occurs after retirement begins. Guyton and Klinger observed that plan failures tended to occur more often later in retirement and were caused by overly generous withdrawals early in the plan. They also identified an insidious but little recognized hazard in plans that did not fail but suffered from reduced purchasing power late in retirement due to an adverse inflationary environment. Guyton and Klinger’s decision rules address all of these hazards.
Some researchers abandoned the constant spending constraint and allowed for variable spending tied to changes in savings valuation; the stock market goes up – spend more, the market goes down – spend less (Pfau 2016b). The difficult part is discovering practical decision rules that adjust annual withdrawals.

Delorme (2016) offered a “Blueprint” for assigning a retirement income management method to a retiree’s preference. His Blueprint combined two withdrawal methods (Constant and Flexible) with two client risk preferences (Safe and Optimal). Delorme’s Safe option protected against plan failure by sacrificing spending in favor of a FTAB surplus. His Optimal option increased disposable income by allowing for variable spending. Delorme offered three computational methods.

- **Constant** was basically Bengen’s 4% Rule with the initial Constant withdrawal amount set to 3.8% of the beginning savings balance. This withdrawal amount was set at beginning of the plan and, with inflation adjustments, retained regardless of anything that happened thereafter.

- **Flexible – RMD+** was an augmented IRS Required Minimum Distribution (RMD) calculation where the annual withdrawal was the savings balance divided by an age related constant from a table supplied by the IRS, plus 2.7%.

- **Flexible – Fixed Percentage (FP)** withdrew 5.6% of remaining savings every year of retirement, adjusted for inflation.

Flexible methods adapted to changing savings balances. They did not suffer from plan failures.

At the core of 3-PEAT is ORP, a **Linear Programming (LP)** application that finds the optimal solution for conflicting requirements. Linear Programming is a mathematical tool where a process is modeled as a set of linear equations. Normally there are more variables than equations. This means there are many solutions to the equations. The objective function is the linear function that spans the entire system and assigns a cost or profit to each variable in the system. This function computes the profit for a solution to the system of equations. The optimal solution is one with the maximum value of the objective function.
The genius of liner programming is the Simplex Method (Dantzig 1963) which evaluates a relatively small number of candidate solutions as it seeks the optimal solution, which is guaranteed to maximize the objective function. (Ragsdale, Cliff T., Andrew F. Seila, and Philip L. Little 1993) and (Coopersmith, Lewis W. and Alan R. Sumutka 2011) demonstrated that the linear model is a suitable representation of the retirement income management process. They both lay out the retirement model in algebraic form. The competing forces of the retirement income model are minimizing taxes (cost reduction) and maximizing compounded asset returns (profit maximization) LP finds the optimal balance between these two. The pivotal constraint is the constant disposable income, index to inflation, assumption.

ORP’s problem definition is opposite that of conventional practice. Other retirement calculators reported in the literature fixed spending at some a priori value and maximized FTAB. They used FTAB to measure the efficiency of competing plans; the larger the FTAB, the better the plan. ORP fixes the FTAB and maximizes disposable income based on the user supplied initial conditions.

Welch (2015) validated the LP approach that fixed FTAB and maximized disposable income. He demonstrated LP’s advantage over the conventional savings withdrawal practice of distributing the taxable account until depletion, then the tax-deferred account, followed by the Roth IRA. His LP approach maximized disposable income by minimizing personal income taxes on tax-deferred withdrawals and other sources of taxable income, while maximizing the compounding of asset returns over the entire planning horizon.

Scott (2017) has a constructed a LP based retirement calculator implemented using the Python programming language. While Scott’s model is a fully formed retirement income management model he concentrates on early retirement issues such as the 10% early withdrawal penalty, the 72(t) exception to the early withdrawal penalty, and the timing of Roth IRA withdrawals. While the experiments reported in this paper were all done using ORP, Scott’s implementation could have served just as well.
LP retirement optimizers are a generation beyond the simulators currently in use. LP optimizers are not only more capable but they offer a “sandbox” for experimentation with various savings withdrawal strategies. Examples are:

1. *Optimizing Distributions from Tax-Deferred Retirement Accounts* (Ragsdale, Cliff T., Andrew F. Seila, and Philip L. Little 1993)
5. *The Impact of Rates of Return on Roth Conversion Decisions and Retiree Savings Wealth* (Coopersmith and Sumutka 2017)

One advantage of LP optimization over simulators is that the IRS 1040 tax table is integrated into the LP model so that the optimizer’s tax efficient results are more relevant to retirement income management.

LP retirement models are examples of the *LP time-dynamic inventory transfer model* (Hirshfeld 1969). Only modest LP formulation skills are required to implement an LP retirement model. There are several modern, commercial or open source LP model generators and solvers available to support such development. Serious research into retirement income management should consider exploration in this direction.

3-PEAT shares basic principles with other variable withdrawal methods. They all compute the withdrawal amount based on remaining assets and time to the end of the planning horizon. The fundamental difference is that 3-PEAT replaces their arithmetic withdrawal functions with a comprehensive
optimization that takes more factors, such as progressive income taxes or delaying Social Security income to age 70, into consideration when computing each year’s withdrawal.

### 3-PEAT Definition

3-PEAT is a manual procedure for managing retirement income. 3-PEAT’s three steps are:

1. The retiree determines her current savings account balance from her monthly brokerage statement, her Social Security benefits from SSA’s annual benefits advisory letter, and her age from her birth certificate.
2. She logs onto the Internet and runs ORP (the LP application that computes her full retirement plan) using these input parameters along with others that provide personal details of her retirement situation.
3. ORP reports an income management plan for the specified retirement time span that shows annual disposable income, and specifies withdrawals for every year of the plan. The retiree withdraws the amount reported as the plan’s first year’s distribution from her savings and budgets her year’s spending to fit the plan’s first year disposable income.

Next year, when the retiree is a year older, she repeats the process, again taking the actions indicated for the first year of her new retirement plan.

The *underlying logic* of 3-PEAT is that ORP, using personal data peculiar to the retiree, projects a retirement income management plan to the planning horizon. The new plan maximizes disposable income, indexed to inflation, in a tax efficient way across the remaining span of retirement. The retiree’s decision making horizon is the current year. Decisions are implemented from computations reported in the first, current, year of the plan. The retiree can be comfortable in making decisions for the current year only, because assumptions about the economic environment one year out can be made with considerably more confidence than those 5 and 10 years out.
Following the aphorism: *Many people say they don’t want to live to be 95, but none of them are 94*, VWS assumes that when the retiree turns age 90 she reconsiders her age 95 planning horizon and increases it to age 96. This is repeated every year until 3-PEAT’s plan ends at age 95. This leaves a modest savings balance at the end of the plan sufficient to support the retiree should she outlive the plan but her FTAB was not depleted nor excessive. From a modeling perspective this avoids a discontinuity at the end of the plan that distorts the final year’s results.

The retiree may have been uncomfortable with the annual withdrawals generated by 3-PEAT if her investments perform poorly, but she is guaranteed to have at least some funds to withdraw right up to the planning horizon. This result holds whatever the retiree’s investment choices as long they can be described to 3-PEAT.

Needless to say this is an idealized description of the process. In practice the retiree makes many runs over several days experimenting with various parameter settings. The step 3 reports are just guidelines for income management. Final cash management decisions will be based on personal preferences and legal factors beyond the scope of the model. ORP just provides a sense of direction for the process.

**Computational Results**

This section summarizes 3-PEAT’s behavior under a variety of conditions. First is a presentation of VWS’s results for Interbellum, 1915-1945. Second is a comparison of 3-PEAT to Delorme’s three withdrawal methods for each of the chosen historic periods.

The retiree being modeled is assumed to be a single, 65 year old with $1M in a tax-deferred account (TDA) and with Social Security income as her only other retirement asset. Her TDA was invested in an index fund that tracked the S&P 500 market index.
100% equity savings (actually an index fund) fund was chosen to bring out the effect of market volatility on savings withdrawals. 3-PEAT’s results for the all equity portfolio serves an upper limit for less, dynamic portfolios. The term **savings** refers to the sum all retirement savings accounts including the TDA, Roth IRA, and taxable accounts.

The retiree’s Social Security income assumes that her **Principle Insurance Amount (PIA)** was $23K. She claimed her benefits at age 70 and enjoyed the 32% delayed starting credit (Pfau 2016b).

3-PEAT’s disposable income is affected by two exogenous factors:

1. **Stock Market Returns**: Changes in savings value, as measured by the change in the S&P 500 stock index in this study, affects the size of the annual savings withdrawal.

2. **Inflation**: Social Security’s **Cost of Living Adjustment (COLA)** affects Social Security income.

**3-PEAT Cash Flow**

This section describes VWS’s results for the Interbellum period as shown in the S&P 500 panel of Figure 1. Interbellum began in 1915, before the U.S. entered World War I and ended in 1945, at the end of World War II. The slope of the line connecting the end points of Interbellum’s market index was a gentle 0.45 but the index was volatile over the 30 years. The average index return was 10.0% and annual return’s standard deviation was 23.4%. Bengen (1994) named two multiyear, severe market downturns in Interbellum. The **Little Dipper** began with the 1929 Stock Market Crash. Bengen named the 1937 sell-off the **Big Dipper**. (Bengen’s naming convention reflected his interest in astronomy.)

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Figure 1
Interbellum 1915-1945

VWS results for Interbellum².
Notes:
1. The S&P 500 panel shows the S&P 500 stock market index for 1915 to 1945, which began with a flat market and included two bull markets and two bear markets.
2. **EndBal** is the annual savings balance showing the effects of both changes in market valuation and 3-PEAT withdrawals.
3. The Cash Flow panel shows where the money is coming from and where it is going.
   a. **SocSec** is Social Security income, beginning at age 70.
   b. **OWA** is the annual Optimal Withdrawal Amount, i.e. the annual withdrawal from TDA.
   c. **Net** represents money withdrawn from the TDA and devoted to DI.
      \[
      \text{Net} = \text{OWA} - \text{Taxes} \\
      \text{Technically taxes should be shared by SocSec and OWA but in the interest of simplicity and without loss of generality this distinction is ignored.}
      \]
   d. **Disposable Income**: \( \text{DI} = \text{Net} + \text{SocSec} \).
4. The EndBal and Cash Flow y-axis is in thousands of nominal dollars.
5. The straight line represents ORP’s constant spending projection from the first ORP run at age 65. The y-axis intercept is 3_PEAT’s $64K initial disposable income and, since it is indexed to 2.5% inflation, it terminates at $133K.

The EndBal panel of Figure 1 graphs TDA annual ending balances across retirement. EndBal’s low point, which occurred during the Little Dipper, was above $700K which left sufficient savings for appreciation during the subsequent economic recovery. EndBal recovered before the Big Dipper began and, coincidently, the end-of-plan liquidation began. At the end of plan (age 95) the TDA retained $437K.
to sustain disposable income to age 101, if needed. This proved to be a sufficient end-of-plan cushion since between 1945 and 1952 the S&P 500 index rose 57%.

The **Cash Flow** panel of Figure 1 shows 3-PEAT’s Net = OWA – taxes was at a high level before age 70 when Social Security income began. Social Security income cut Net, and OWA, in half. The early period market was stable and Net grew gradually until the Roaring 20s when growth accelerated. The 1929 crash contracted Net.

The initial ORP run at the beginning of retirement, with 6% rate of return and 2.5% inflation, projected total disposable income over all of retirement (with zero FTAB) at $2,927K while 3-PEAT’s total disposable income plus FTAB was $3,736K.

Net’s low point at $46K was not during depression but came in 1943. Thanks to the Social Security COLA DI’s low point did not come during the depression.

Appendix A demonstrates the use of VWS to perform the calculations that resulted in Figure 1.

**Comparison of Variable Withdrawal Methods**

The following section compares 3-PEAT to three of Delorme’s (2016) four savings withdrawal methods as described earlier in the Literature Review. Delorme’s *Optimal and Constant* variation, which is the 4% Rule with a 5.4% initial withdrawal rate, was omitted. *Safe and Constant* turned out to be unsafe. Hence, *Optimal and Constant* did not offer any new information.

**Comparing Withdrawal Methods**

The following section compares DI and EndBal for the four withdrawal methods. Interbellum was chosen because of its two traumatic market events. This discussion is an extension of the 3-PEAT results presented earlier by Figure 1.
Figure 2
VWS Comparison of the Four Withdrawal Methods’ DI and EndBal for Interbellum

- **EndBal**: End of the year TDA balance.
- **DI**: Disposable income is withdrawal minus income tax.
- Figure 1 showed the graph of the S&P 500 for Interbellum.
- All retirement income was from TDA withdrawals. Social Security income was not modeled.
- All methods applied the income tax rate computed by ORP.
- The y-axis is thousands of nominal dollars.

Before the Little Dipper, Figure 2 gives the illusion that all variable withdrawal methods were approximately the same. This is partially because the y-axis is scaled to the 1928 bubble which obscures volatility before the Roaring Twenties.
• The 4% Rule’s DI had low variability, fluctuating according to the CPI, but its EndBal was as volatile as the others. While the 4% Rule under withdrew and left a ridiculous FTAB ($2.4M), it did not fail during Interbellum.

• Withdrawing too little left FP with a large FTAB ($757K). Toward the end of the plan FP’s large EndBals and a rising market increased FP’s DI.

• The RMD is designed by the IRS to drive the TDA balance toward zero so that the government can collect their deferred taxes. At mid plan RMD+’s EndBal goes into descent as a result of its algorithm. By age 95 RMD+’s EndBal is nearly zero. As RMD+’s EndBal declined so did its DI.

• 3-PEAT directed EndBal toward zero, thanks to ORP’s zero FTAB constraint. Near Interbellum’s end 3-PEAT’s EndBal leveled off as the retiree extended her plan horizon each year after age 90. 3-PEAT did not have the highest DI before the Little Dipper but, because of its smaller withdrawals during that period, 3-PEAT’s DI exceeded the others during the Big Dipper.

Interbellum ended with a recovering stock market. The four methods responded to the recovery with four different performances.

To address Figure 2’s illusion that all variable withdrawal methods are alike the next section compares the wealth added to retirement income by the four methods.

**Measuring Plan Worth**

The economic worth of a withdrawal method can be measured by its total DI over the duration of the plan. Table 1 compares the DIs for the four methods across the six time periods.

<table>
<thead>
<tr>
<th>Table 1</th>
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</thead>
<tbody>
<tr>
<td>Plan Value Comparison</td>
</tr>
<tr>
<td>Compare the total DI of the withdrawal methods over the six time periods.</td>
</tr>
</tbody>
</table>
The 4% Rule suffered two plan failures. FP and 3-PEAT are immune to plan failure. Table 1 shows that the plans’ results are sensitive to the method used to compute withdrawals. The 4% Rule’s total withdrawals during the Modern Era suffered from a combination of a strong stock market, which it ignored, and low inflation which held down its annual DI increases.

The Stagflation period was a difficult period in which to be retired. The initial savings balance in 1973 was one million dollars but only $985,000 (real) was available for disposable income. The remainder went to pay taxes. All other periods distributed substantially more than the initial balance out as disposable income. Recall that there is no Social Security income for these test and that the values in Table 1 are real, i.e. the effects of inflation have been removed.

**Measuring Plan Variability**

A plan’s psychological worth, i.e. making the retiree comfortable with withdrawing from her retirement savings, can be measured by how much DI variability she has to endure. The retiree’s expectation is that
her initial DI is indicative of her annual DI throughout retirement. It becomes a source of concern when her withdrawal method reduces a year’s DI below her initial DI.

Table 2 offers two measures of method variability.

<table>
<thead>
<tr>
<th>Panel A* (%)</th>
<th>Modern Era</th>
<th>Big Bang</th>
<th>Stagflation</th>
<th>Interbellum</th>
<th>Worst Case</th>
<th>Gilded Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-PEAT</td>
<td>0.0</td>
<td>-29.4</td>
<td>-53.5</td>
<td>-38.6</td>
<td>-50.8</td>
<td>-17.2</td>
</tr>
<tr>
<td>4% Rule</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>RMD+</td>
<td>-61.9</td>
<td>-82.4</td>
<td>-79.8</td>
<td>-71.6</td>
<td>-82.2</td>
<td>-58.8</td>
</tr>
<tr>
<td>FP</td>
<td>-30.1</td>
<td>-61.6</td>
<td>-71.1</td>
<td>-52.6</td>
<td>-71.0</td>
<td>-32.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B** (Years):</th>
<th>Modern Era</th>
<th>Big Bang</th>
<th>Stagflation</th>
<th>Interbellum</th>
<th>Worst Case</th>
<th>Gilded Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-PEAT</td>
<td>0</td>
<td>8</td>
<td>30</td>
<td>10</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>4% Rule</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RMD+</td>
<td>7</td>
<td>30</td>
<td>28</td>
<td>16</td>
<td>19</td>
<td>8</td>
</tr>
<tr>
<td>FP</td>
<td>5</td>
<td>30</td>
<td>30</td>
<td>23</td>
<td>26</td>
<td>21</td>
</tr>
</tbody>
</table>

Notes:
*Panel A is the percent difference between the lowest DI and the initial DI relative to the initial DI, i.e. = (min(DI) – initial DI)/initial DI.
**Panel B shows the number of years that each method’s DI was below its initial DI.

- DI values were real amounts, i.e. inflation was removed.
- A blank entry indicates a plan failure for that period.
- 0.0% indicates that the initial DI was the smallest DI for each period. 4% Rule initial entries were the lowest by definition.
- There is no Social Security income since the other methods do not model it.

In other words, Panel A of Table 2 is the retirees discomfort index. The more negative the Panel A percentage the more the discomfort and the larger the Panel B count the longer the discomfort lasts.

Some researchers (Kitces 2012) separate retirement spending into:
1. **Essential**: expenses that have to be met such as food, shelter, and clothing.

2. **Discretionary**: optional spending that can be foregone, such as cruises or golf.

Panel A’s message is that if essential spending exceeds 50% of disposable income then the prospective retiree is not financially ready for retirement.

In summary, 3-PEAT produced the highest total DI and lowest DI downside variability.

**3-PEAT in Real Life**

At the beginning of each year 3-PEAT reports OWA as a single value. The married retiree decides whether to withdraw from her tax-deferred account, her spouse’s account or both; the choice does not matter to 3-PEAT. The only requirement is that their two separate RMDs are met.

The 3-PEAT user enjoys a certain amount of latitude in deviating from 3-PEAT’s plan:

1. Withdrawals from the wrong account may divert the plan from the optimal path.

2. Withdrawing more than 3-PEAT’s recommended amount for essential spending is really stealing from the future. If done with discretion this will reduce total DI but if done in excess will deplete EndBal -- which is plan failure.

3. Withdrawals that are less than the recommended amount leaves larger account balances for the **lean** years. If the withdrawals are less than the RMD then the difference is transferred to the taxable account.

The retiree’s withdrawals will be reflected in the initial account balances for the forth-coming year (3-PEAT’s Step 1).

TDA withdrawals are assumed to take place at the first of the year. In fact they can take place anytime during the year with little effect on the plan’s overall outcome.
Conclusion

3-PEAT is a practical, safe procedure for managing income over the term of retirement. 3-PEAT avoids plan failures caused by premature savings depletion. As compared to other variable withdrawal methods, 3-PEAT increases disposable income, and reduces income variability. 3-PEAT’s computed disposable income varies from year to year because of variable savings withdrawals, changes in asset market valuation, and inflation induced changes in Social Security benefits. This is a fact of life for retirees.

Given that the resulting actual withdrawal profile cannot be fully known until the end of the retirement 30 years hence, 3-PEAT is probably the best the retiree can do given zero foresight. Not only does 3-PEAT immunize a retirement savings plan from plan failure, it increases the total amount of disposable income relative to other withdrawal methods. Although 3-PEAT increases disposable income over the planning period, it does not necessarily maximize it. There may be even better plans with larger total disposable income.

Retirees face a psychological challenge as they enter retirement. They may have considerable angst when retirement savings stop growing and begin to be consumed. 3-PEAT can be an important arrow in the financial advisor’s quiver to aid in helping clients adjust to consuming savings. This paper has shown that 3-PEAT is a savings withdrawal method that will not prematurely deplete retirement savings and will reduce year-to-year disposable income variability. The do-it-yourselfers that were observed practicing their own form of 3-PEAT were known to be engineers and accountants, a population that in many cases already had some acquaintance with linear programming and were comfortable with quantitative methods. This leaves a rather large population of retirees that financial advisors can comfort with the assurance that 3-PEAT is mathematically sound and has been shown to not have deserted them during known difficult market conditions. The fearful can be assured that the math is on their side.
References


Appendix A: Variable Withdrawal Simulator (VWS) Example

In this appendix VWS plays the role of the retiree as it compresses 30 years of retirement into Table 3 to demonstrate the repeated use of 3-PEAT over the Interbellum 1915-1945 period.

VWS is a spreadsheet to be filled in manually by whatever poor, unfortunate sot who can be recruited to make 30 optimization runs with systematic changes in the initial conditions. VWS is a proof of a concept to assure retirees that what they are doing is safe. VWS is not a product for the retail market. All of the results in this article were computed in this manner.

VWS proceeds as follows:

VWS user logs on to ORP (www.i-orp.com) using whatever browser that suits them. The long version of the parameter form is selected. The retiree’s current age is set to 65, the Tax-deferred account initial balance is set to 1,000 (i.e. one million dollars), the Social Security PIA is set to 23 (i.e. 23 thousand
dollars) beginning at age 70, and the plan end is set to age 95 to provide for the conventional 30 year retirement. ORP’s default settings are used for all other parameters. These initial conditions are shown on the left side of Table 3. The Run ORP button is clicked. The first line of ORP’s first two reports provide the computed values on the right side of Table 3.

<table>
<thead>
<tr>
<th>Age</th>
<th>Year</th>
<th>S&amp;P %</th>
<th>EndBal</th>
<th>SocSec</th>
<th>DI</th>
<th>BegBal</th>
<th>RMD</th>
<th>OWA</th>
<th>Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>1915</td>
<td>-4.9%</td>
<td>1,000</td>
<td>64</td>
<td>925</td>
<td>0</td>
<td>75</td>
<td>12</td>
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<tr>
<td>66</td>
<td>1916</td>
<td>30.7%</td>
<td>1,209</td>
<td>73</td>
<td>1,121</td>
<td>0</td>
<td>88</td>
<td>15</td>
<td></td>
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<tr>
<td>67</td>
<td>1917</td>
<td>9.8%</td>
<td>1,231</td>
<td>76</td>
<td>1,139</td>
<td>0</td>
<td>92</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>68</td>
<td>1918</td>
<td>-16.8%</td>
<td>948</td>
<td>67</td>
<td>868</td>
<td>0</td>
<td>80</td>
<td>13</td>
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<tr>
<td>69</td>
<td>1919</td>
<td>15.6%</td>
<td>1,004</td>
<td>71</td>
<td>918</td>
<td>0</td>
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<tr>
<td>70</td>
<td>1920</td>
<td>18.3%</td>
<td>1,086</td>
<td>37</td>
<td>1,024</td>
<td>37</td>
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<td>839</td>
<td>32</td>
<td>52</td>
<td>14</td>
<td></td>
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<tr>
<td>72</td>
<td>1922</td>
<td>9.7%</td>
<td>920</td>
<td>37</td>
<td>865</td>
<td>34</td>
<td>55</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>1923</td>
<td>27.9%</td>
<td>1,106</td>
<td>37</td>
<td>1,038</td>
<td>42</td>
<td>68</td>
<td>18</td>
<td></td>
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<tr>
<td>74</td>
<td>1924</td>
<td>5.4%</td>
<td>1,094</td>
<td>38</td>
<td>1,025</td>
<td>43</td>
<td>69</td>
<td>18</td>
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<tr>
<td>75</td>
<td>1925</td>
<td>25.5%</td>
<td>1,286</td>
<td>38</td>
<td>1,202</td>
<td>52</td>
<td>84</td>
<td>22</td>
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<tr>
<td>76</td>
<td>1926</td>
<td>25.0%</td>
<td>1,503</td>
<td>39</td>
<td>1,402</td>
<td>64</td>
<td>101</td>
<td>27</td>
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<tr>
<td>77</td>
<td>1927</td>
<td>11.7%</td>
<td>1,566</td>
<td>39</td>
<td>1,456</td>
<td>69</td>
<td>110</td>
<td>30</td>
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<tr>
<td>78</td>
<td>1928</td>
<td>35.7%</td>
<td>1,975</td>
<td>39</td>
<td>1,831</td>
<td>90</td>
<td>144</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>79</td>
<td>1929</td>
<td>45.7%</td>
<td>2,668</td>
<td>39</td>
<td>2,465</td>
<td>126</td>
<td>203</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>1930</td>
<td>-8.2%</td>
<td>2,264</td>
<td>39</td>
<td>2,084</td>
<td>111</td>
<td>180</td>
<td>50</td>
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<tr>
<td>81</td>
<td>1931</td>
<td>-21.3%</td>
<td>1,641</td>
<td>39</td>
<td>1,504</td>
<td>84</td>
<td>137</td>
<td>37</td>
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<tr>
<td>82</td>
<td>1932</td>
<td>-42.0%</td>
<td>872</td>
<td>39</td>
<td>795</td>
<td>47</td>
<td>77</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>83</td>
<td>1933</td>
<td>-8.4%</td>
<td>728</td>
<td>39</td>
<td>660</td>
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<td>68</td>
<td>18</td>
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<tr>
<td>84</td>
<td>1934</td>
<td>52.9%</td>
<td>1,009</td>
<td>40</td>
<td>909</td>
<td>59</td>
<td>100</td>
<td>27</td>
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<tr>
<td>85</td>
<td>1935</td>
<td>-7.1%</td>
<td>845</td>
<td>42</td>
<td>755</td>
<td>51</td>
<td>90</td>
<td>25</td>
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<tr>
<td>86</td>
<td>1936</td>
<td>53.8%</td>
<td>1,161</td>
<td>42</td>
<td>1,027</td>
<td>73</td>
<td>134</td>
<td>37</td>
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<tr>
<td>87</td>
<td>1937</td>
<td>32.4%</td>
<td>1,360</td>
<td>43</td>
<td>1,188</td>
<td>89</td>
<td>172</td>
<td>49</td>
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<tr>
<td>88</td>
<td>1938</td>
<td>-31.2%</td>
<td>817</td>
<td>43</td>
<td>702</td>
<td>55</td>
<td>115</td>
<td>32</td>
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<tr>
<td>89</td>
<td>1939</td>
<td>15.5%</td>
<td>811</td>
<td>43</td>
<td>683</td>
<td>57</td>
<td>128</td>
<td>36</td>
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<tr>
<td>90</td>
<td>1940</td>
<td>3.8%</td>
<td>709</td>
<td>43</td>
<td>597</td>
<td>52</td>
<td>112</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>1941</td>
<td>-7.5%</td>
<td>552</td>
<td>44</td>
<td>465</td>
<td>43</td>
<td>87</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>92</td>
<td>1942</td>
<td>-8.7%</td>
<td>424</td>
<td>49</td>
<td>357</td>
<td>35</td>
<td>67</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>93</td>
<td>1943</td>
<td>19.0%</td>
<td>425</td>
<td>53</td>
<td>358</td>
<td>37</td>
<td>67</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>94</td>
<td>1944</td>
<td>22.8%</td>
<td>440</td>
<td>54</td>
<td>371</td>
<td>41</td>
<td>69</td>
<td>22</td>
<td></td>
</tr>
</tbody>
</table>
Notes:
1. **Age** is the age of the retiree. Age, not year, is the index for Social Security income.
2. **Year** is the year of Interbellum to which the values apply.
3. **S&P%** is the percent year-to-year change in the S&P 500 stock market index. (Shiller 2016).
   See the Glossary for how the S&P 500 annual return is computed.
4. **EndBal** is the ending TDA balance on the last business day of the previous year. EndBal is the product of the previous year’s BegBal (account balance after withdrawals) and the percent change in the S&P 500 market index for the year.
5. **SocSec** is the amount of annual Social Security income. Benefits are attached to age and not to historical year. Claiming benefits at age 70 provides for a 32% premium over the $23,000 PIA.
6. **DI** is the computed disposable Income for the coming year. DI = OWA + SocSec – Taxes.
7. **BegBal** is the TDA balance at the start of the year after the annual withdrawal. Next year’s EndBal is this year’s BegBal times the percent change in the S&P 500.
8. **RMD** is TDA’s Required Minimum Distribution.
9. **OWA** is the amount withdrawn from the TDA to support DI.
10. **Taxes** are estimated income taxes for the year based on TDA distributions and Social Security benefits. 3-PEAT uses the IRS 2016 estimate tax return tables in this computation.

Columns 1-3 of Table 3 are from Shiller’s data. Columns 4 and 5 are computed by VWS. The rest of the columns are computed by ORP.

Amounts are thousands of nominal dollars.

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On January 1, 1915, the 30-year retirement is planned. The probability of the retiree actually living to age 95 is small, but for longevity safety’s sake that was her planning horizon.

Table 3 shows how VWS worked from age 65 through age 95. On January 1, 1915, at age 65, the simulated retiree ran ORP. Her input parameters were:

1. **EndBal**: $1M was TDA’s beginning balance as of December 31, 1914.
   Thereafter VWS computed EndBal as: \( \text{EndBal} = \text{BegBal}_{t-1} \times r_y \) where \( r_y \) was the percent change in the S&P 500 stock market index for year “\( y \)”. This computation is supplied by the VWS spreadsheet.
2. **SocSec**: Social Security income began at age 70.
The ages of 70 and 90 are highlighted because they include shifts in initial conditions. At age 70 Social Security income is initiated and the RMD begins. The initial age 70 Social Security income is computed by applying compounded Social Security COLA (Shiller 2016) to the $23K PIA and adjusting it upward with the 132% delay claim premium. This corresponds to the Social Security Administration’s annual advisory letter.

At age 90 and thereafter the planning horizon is incremented along with the current age because the simulated retiree becomes fearful that she will outlive her savings.

From each ORP report the user retrieved the five values found on the right side of the first line of Table 3:

1. **DI**: $64K was the disposable income (DI) for 1915. DI was the amount available for spending throughout the year.

2. **BegBal**: $925K was the remaining TDA balance after the year’s withdrawal (OWA = $75K) had been debited, as of January 2, 1915. This corresponds to the amounts on the brokerage statement.

3. **RMD**: The TDA’s required minimum distribution began at age 70.

4. **OWA**: $75K was the TDA’s Optimal Withdrawal Amount.

5. **Taxes**: $12K was the estimated Federal income tax for the year.

EndBal and SocSec are normally provided by the retiree. For simulation purposes VWS computed them as columns 3 and 4 of Table 3.

3-PEAT’s critical step was to compute OWA for the current year. The plan’s FTAB was fixed at zero and ORP computed the maximum DI and its supporting OWA for an age 95 planning horizon. (The choice of a zero FTAB was arbitrary. A larger FTAB can provide a bequest or a financial buffer at plan’s end as savings cushion in case the retiree outlives her plan.)
On January 1, 1916 the now 66 year old retiree computed her plan for the second year of retirement. Table 3 shows that in 1915 the S&P500 index plus dividends rose 30.7%. BegBal shows that the 1915 beginning balance was $1,121K after the $73K withdrawal. Consequently VWS increased the 1916 TDA balance (EndBal) from $1,000K to $1,209K (EndBal, line 2). (In actual practice the monthly brokerage statement provided EndBal.) The simulated retiree withdrew $75K (OWA) from her savings, allocated $12K for taxes, and budgeted $64K (DI) for the year’s spending.

Withdrawals from the TDA (OWA=$75K) paid taxes and filled the gap between DI ($64K) and other income sources (SocSec = $0 for the first year). In theory DI = OWA + SocSec − Taxes. In practice rounding error caused a $1K discrepancy. ORP computes in pennies and reports thousands of dollars.

The simulated retiree made the ORP run for year 2 with a beginning TDA balance (BegBal) of $1,209K and with a 29 year planning horizon ending to age 95. The 1916 decision variables are harvested from ORP:

1. **OWA:** $88K (1916, OWA) is withdrawn from the TDA, up from the previous year’s $75K, that resulted in a 1916 BegBal of $1,121K.

2. **DI:** $73K was the DI for 1916 up from 1915’s $64K. The $73K DI was the after tax amount that was available for spending during 1916.

During 1916 the S&P 500 index rose 9.8% and the TDA balance (EndBal), driven by market forces, increased to $1,231K on December 31, 1916.

At age 70 Social Security benefits began along with the TDA’s RMD. After age 70 Social Security income is input into ORP from SSA computed schedules.

Beginning at age 90 the planning horizon age is incremented along with current age.
This process was repeated for each year of simulated retirement until age 95. The plan’s FTAB was $440K in nominal dollars.

The requirements of simulation make the application of VWS more complicated than the annual use of 3-PEAT. All steps are computed in nominal dollars because that is what the retiree sees during the application of 3-PEAT.

The VWS spreadsheets for this study may be downloaded from www.i-opr.com/3-PEat/3-PEATvws.xlsx.
**Glossary**

**BegBal:** The savings balance at the beginning of the year after the annual withdrawal but before the year’s change in valuation has been applied.

**COLA:** Social Security Cost of living adjustment as defined by the Social Security Administration.

**CPI:** Consumer Price Index from Shiller’s data.

Disposable Income or **DI:** Disposable Income is the after-tax money available for personal consumption in retirement. Spending is what the retiree wants to do. Disposable income is a computed limit on what the retiree should do.

**EndBal:** TDA’s ending balance for each year of retirement. EndBal is computed by VWS from S&P 500 historical data and BegBal.

**FP:** The Fixed Percentage withdrawal method annually withdraws a fixed percentage of the savings balance. This study used Delorme’s (2016) recommended 5.6%.

**FTAB:** The Final Total Asset Balance is the sum of all retirement savings accounts (tax-deferred, Roth IRA, and taxable) at the final year of the plan. The final EndBal at age 95 is the plan’s FTAB.

**LP:** Linear Programming is a mathematical modeling technique that computes the optimum for conflicting requirements. LP describes a modeling situation with a set of linear equations. In retirement planning LP maximizes disposable income by computing a tax-efficient solution that maximizes return of retirement assets.

**Net:** The net annual withdrawal from savings after tax has been deducted. Net is the withdrawal contribution to disposable income.

**Nominal** amounts are dollar amounts recorded at the time of the event. See Real amounts.
**OWA:** The Optimal Withdrawal Amount is 3-PEAT’s annual retirement savings withdrawal.

**ORP:** The Optimal Retirement Planner is an Internet based, linear programming application that computes the optimal retirement plan; a tax-efficient maximization of disposable income. ORP can be found at www.i-orp.com at no charge, no registration, no advertisements, and no download requirements. ORP is offered as a community service to acquaint the retirement naïve with some of the issues of retirement income management. Secondly the site demonstrates how an LP model can be made available to the general public who are not operations research practitioners.

**PIA:** The retiree’s Principle Insurance Amount is computed by the Social Security Administration. It is the amount of annual income due the retiree at their full retirement age, most likely age 67, based on their work history and paid in payroll taxes.

Real amounts are the nominal amounts with inflation, relative to the beginning of the historical period, removed. Real and nominal amounts are the same for the first year of the historical period but thereafter they diverge.

**RMD:** The TDA’s Required Minimum Distribution as defined by the IRS.

**RMD+:** Delorme’s savings withdrawal method based on the RMD whereby withdrawals are based on the IRA table of constants increased by Delorme’s recommended 2.7%.

**Savings:** All retirement savings accounts including the TDA, Roth IRA, and taxable accounts.

Spending: What the retiree chooses to consume within the limit of 3-PEAT’s disposable income.

**S&P 500%:** The percent year to year change for the S&P 500 stock market index including the year’s dividends. \( S&P\text{ }500\% = \frac{(S&P500_y - S&P500_{y+1})}{S&P500_{y+1}} + \text{Dividends}_y/S&P500_y \), where \( y \) is the year.
The first term is the percent change in value in the index and the second term is the dividend as a percentage of the ending of period index.

**SocSec:** The VWS’s Social Security income for each year of retirement.

**Taxes:** Personal income taxes on 3-PEAT’s TDA distributions and 85% of Social Security benefits.

**TDA:** Tax-deferred account such as a traditional IRA, 401K, and 403B. Taxes on contributions are deferred until withdrawal.

**Variable Withdrawal Simulator (VWS):** simulates the actions of a retiree using 3-PEAT to execute a retirement plan.

Interbellum is commonly defined as the period between the end of World War 1 (1918) and the beginning of World War 2 (1939). In order to get a 30 year retirement span the two world wars are included in Interbellum in this paper.

2 Additional results, including cash flow graphs for all eras, can be found at www.i-orp.com/3-PEAT/3-PEATsupplement.pdf.

3 Floor and Ceiling withdrawal tactics violate 3-PEAT’s program in the interest of smoothing savings withdrawals and meeting essential spending requirements. This is discussed more fully in a supplemental report to be found at i-orp.com/3-PEAT/FandC.pdf.