Abstract

A fundamental assumption of retirement financial planning is to manage withdrawals from retirement savings and other sources of income so as to maintain the maximum, constant, inflation adjusted, after tax, stream of money available for spending.

- This requires a strategy for maximizing the annual, after tax, withdrawals from tax-advantaged retirement savings.
- Money available for spending is increased by minimizing taxes on tax-deferred account withdrawals.
- Taxes are minimized by transferring some tax-deferred withdrawals into other accounts during periods of low taxes.
- Later in the plan withdrawals from the other accounts supplement tax-deferred withdrawals to maintain a constant rate of spending.
- Linear Programming (LP) is a useful method for computing such a strategy.

The essence of the optimal plan is to optimize retirement account withdrawals and transfers to minimize personal income taxes on tax-deferred withdrawals.

In this paper retirement savings account withdrawal plans computed by a linear programming optimizer are compared to the ubiquitous, conventional wisdom strategy to measure the degree of improvement that the optimized plan offers.

1 Introduction

This paper addresses the question of: “How much does a Linear Programming (LP) application improve on the conventional wisdom for withdrawing funds from tax-advantaged retirement savings accounts?”

LP is an Operations Research tool that has been a successful computer application since the late 1950’s [22]. One useful LP application is to maximize the funds available for spending from annual tax-advantaged retirement saving account withdrawals.

The essential element of the optimal plan is that when the tax situation is favorable money is withdrawn from the tax-deferred account and transferred to other types of accounts and later those accounts supplement tax-deferred withdrawals to meet spending requirements.

2 Background

The three different types of retirement savings accounts are:
1. **Tax-deferred (IRA):** There are no income taxes on employment earnings contributed to the IRA but all withdrawals are taxed as personal income. This type of account includes IRAs, 401k, 403b and a variety of others, all of which are generically the same. The term IRA will be used herein to denote tax-deferred accounts since most of the others are rolled over into an IRA before or at retirement. IRAs have a **Required Minimum Distribution (RMD).** The RMD is an amount that the IRS requires be withdrawn annually beginning at the age of 70½. It is computed as the IRA account balance on January 1 divided by a life expectancy value taken from an IRS published table. [15].

2. **Roth IRA:** Income taxes are paid on the employment income contributions to a Roth IRA but there are no taxes on withdrawals. In addition to employment contributions withdrawals from an IRA may be transferred into a Roth IRA after personal income taxes have been paid on the withdrawals.

3. **After-tax account:** Contributions to the After-tax account can be from any source, after taxes have been paid. Taxes are paid on sales, dividends and interest as they are incurred. No taxes are paid on withdrawals. After tax IRA withdrawals may be transferred into the After-tax account.

From the perspective of how they are taxed, these accounts are three entirely different entities.

After the retiree reaches the age of 59 1/2 withdrawals can be made from any account in any amount without penalty. The amount of withdrawals and the order in which they are made (the withdrawal plan) will affect the amount of money available annually, after taxes, for spending over the term of retirement. The spending for different withdrawal plans will vary because of the Federal progressive income tax and the RMD.

The goal of a withdrawal plan is to maximize spending. The relative efficiency of one plan over another is the percentage improvement of the plan with the larger spending to the smaller.

The **conventional wisdom** offered by many financial advisers and recommended by the financial press is:

1. draw down the After-tax account first,
2. draw down the IRA second,
3. draw down the Roth IRA last.

Raabe and Toolson [7] showed that the conventional wisdom is more efficient than any other permutation of serial account distributions.

The conventional wisdom is not tax-efficient because it pays no income taxes either during the first distribution phase when the After-tax account is being drawn down or during the third phase when the
Roth IRA is being drawn down. This compresses IRA taxable withdrawals into the middle phase which will likely push part of these distributions into a higher income tax bracket than if they were spread over the entire retirement term.

3 The Experiment

Two computer programs were used to compare the efficiency of Linear Programming (LP) optimizer to the conventional wisdom:

1. The **Optimal Retirement Planner (ORP)** [8] is the LP application that was used to compute the optimal plans. ORP accepts a model consisting of a set of activities that can be done and constraints on those activities. It then computes a solution to the model that has a maximum economic advantage [6]. ORP maximizes spending for each year of retirement, after income taxes are paid. LP mathematically guarantees that there is no better solution than the one computed [16].

2. The **Conventional Wisdom Simulator (CWS)** is a spreadsheet that simulates the conventional wisdom over the term of retirement. CWS was implemented for the purpose of providing a benchmark to measure the efficiency of ORP generated plans.

The two programs use the same parameter set and compute to the same objective; maximize spending over retirement. Spending for age 65 is in today’s dollars and spending for subsequent years are subjected to compounded inflation. The value of a computed plan is measured by spending at age 65 which is in today’s dollar.

Given a set of parameters the program’s objective is to find the highest spending that will not deplete retirement funds before the end of the plan (no plan failures) and will leave a zero balance in the estate (no unspent surplus). Both programs annually reduce the IRA balance by the IRA withdrawal plus taxes paid. This means that the IRA balance goes down by more than just the withdrawal for spending. Taxes were computed using 2011 IRS tables. For a given set of parameters, spending computed by the two programs can be compared and the more efficient plan can be judged.

The situation being modeled is a 65 year old, unmarried retiree who saved $1,000,000 for retirement. The IRA contains $400,000, the Roth IRA $350,000 and the After-tax account $250,000. These proportions were arrived at by running ORP for a 30 year old and evaluating the plan value at age 65. The Roth IRA amount is lower than the IRA because of income taxes paid on the Roth IRA contributions. The After-tax amount is lower yet because not only were income taxes paid on contributions but 15% capital gains taxes were paid on annual returns.

Other assumptions are, unless specific to a given scenario, a 2.5% inflation rate and a 5% investment rate of return (ROR).
4 Computational Results

The experiment was to run the two programs for the parameter set described above and to compare the efficiency results. Then the ORP detail results are examined to understand the dynamics of the solution. This process is repeated for additional scenarios created by including an additional feature in the base scenario.

4.1 Single ROR

The simplest scenarios to explore are those in which all accounts have the same ROR. This allows ORP to clearly demonstrate the consequences of personal income tax minimization. Later, in section 4.2, different ROR’s for different accounts will introduce more complex economics.

4.1.1 Base Scenario

The Base Scenario is for the parameter set described above. Table 1 reports spending in today’s dollars, computed by CWS and ORP, for different rates of investment return.

<table>
<thead>
<tr>
<th>ROR</th>
<th>CWS Spending</th>
<th>CWS Tax Rate</th>
<th>ORP Spending</th>
<th>ORP Tax Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>$36,600</td>
<td>10%</td>
<td>$41,000</td>
<td>7%</td>
<td>12%</td>
</tr>
<tr>
<td>5</td>
<td>42,600</td>
<td>11</td>
<td>46,000</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>46,000</td>
<td>11</td>
<td>53,000</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>53,200</td>
<td>12</td>
<td>59,000</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>57,700</td>
<td>13</td>
<td>66,000</td>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 1: Comparison of CWS to ORP

ROR is the rate of investment return; the parameter that was varied for the results in this table. The Spending columns show the plan’s spending for the first year of retirement in today’s dollars for the different values of ROR. The Tax Rate columns contain the effective personal income tax rate on IRA withdrawals. It is computed as taxes paid on an IRA withdrawal divided by the IRA withdrawal for age 75. Age 75 was judged to be more or less representative of both plans’ withdrawals. The Efficiency column shows the advantage of the ORP withdrawal plan over CWS with the spending difference as a percentage of CWS spending:

\[(\text{ORP spending} - \text{CWS spending}) / \text{CWS spending}\].

Figure 1 demonstrates the dynamics of the ORP withdrawal plan for the 5% ROR. It shows how money is withdrawn from the three retirement accounts over the term of retirement.
Withdrawals from the IRA (blue) stay in a relative narrow ban throughout retirement. This keeps income taxes in a low tax bracket.

Until age 70 the spending line (purple) and the After-tax withdrawal line (red) are the same (top line) because spending is being funded exclusively from the After-tax account. IRA withdrawals during this period are rolled over into the Roth IRA account. At age 70 the RMD kicks in, increasing the required withdrawals from the IRA and correspondingly lowering the withdrawals from the After-tax account. RMD money must be spent because, by law, it cannot be rolled over into a Roth IRA. At age 71 the After-tax account is depleted and withdrawals begin from the Roth IRA (green). At age 71 spending is partially funded by all three accounts. After age 71 the After-tax account is depleted and parallel withdrawals from the IRA and Roth IRA fund spending. At age 91 IRA withdrawals begin to rise while the Roth IRA begins to tail off. This is because IRA withdrawals are meeting more of the spending requirements. At age 95 both accounts are taken to zero. The ORP strategy of delaying some IRA withdrawals until the end is apparent in the large withdrawal in the final year which is offset by a corresponding reduced Roth IRA withdrawal.

Figure 2 shows IRA account activity during retirement and gives a perspective as to what is driving the IRA distributions.
The blue line represents IRA distributions. The red line, to the left, represents IRA to Roth IRA transfers. The RMD level (green) is fixed by law according to the balance of the IRA and not subject to optimizer modification. After age 70 IRA distributions are being constrained by the RMD until age 91. The IRA distributions are seeking a lower level but it cannot go below the RMD. Because of the way it is defined [15] the RMD remains remarkably flat for most of retirement. Looking back at Figure 1 it is seen that as the RMD, thus IRA withdrawals, decline the Roth IRA is filling in to meet spending requirements. At age 91 spending needs are large enough such that the RMD no longer constrains IRA withdrawals.

During most of the plan ORP would prefer to distribute less from the IRA and more from the Roth IRA. Were it not for the RMD the blue line would continue at the initial level of the red line well into retirement. Clearly the RMD is costing retirees money although how much is yet to be explored.

It might be the case that since the IRA requires that the RMD be calculated using a life expectancy in the 90’s that shortening the retiree’s specified life expectancy to 85 would raise the IRA withdrawals above the RMD. Experiments showed that the IRA withdrawals stays fixed to the RMD.

Figure 3 shows how IRA withdrawals fit into the Federal progressive income tax brackets.
Each vertical bar represents IRA withdrawals for one age. For example, the age 65 bar shows $20,000 worth of withdrawals, divided into the No Tax and 10% brackets. The No Tax bracket excludes $11,000 of withdrawals because of the standard deduction and one personal exemption.

Before the age of 70 IRA withdrawals are at the upper bound of the 10% bracket but no more. After the RMD starts up withdrawals are forced into the 15% bracket. Looking back at Figure 1 and Figure 2, the IRA withdrawals stay with the RMD all the way to age 91. There all IRA withdrawals are in the No Tax bracket until the final year when the final large IRA withdrawal returns to the 10% bracket.

Figure 4 shows the annual asset balances of the three accounts which reflect the withdrawals shown in Figure 1.
The After-tax account decreases to zero from the beginning of retirement to age 73 while it is being drawn down for spending. Before the age of 70 the IRA increases in value since transfers to the Roth IRA are smaller than its asset returns (5%).

At age 75, when Roth IRA withdrawals begin, the balance of the Roth IRA is significantly larger than the IRA because the IRA started distributions 5 years earlier and because of the transfers from the IRA to the Roth IRA.

Figure 5 shows the account balances computed by CWS. It compares to the ORP account balances shown in Figure 4.
CWS’s strategy it to spend down the retirement accounts serially. From ages 65 to 70 the After-tax account is used for spending and the IRA and Roth IRA accounts compound in an identical fashion. From age 70 to 75 the RMD requires IRA distributions even though there is still After-tax money available for spending. After-tax withdrawals proceed at a reduced rate because the RMD withdrawals from the IRA are now a part of spending. When the After-tax account is depleted spending is only from the IRA while the Roth IRA continues to appreciate. After the IRA is depleted then spending is funded by the Roth IRA to the end of the plan.

4.1.2 Social Security Benefits Scenario
Table 2 shows the base scenario with $23,000 of annual Social Security benefits added to the retirement plan. The Social Security demonstrates the impact of a steady stream of income over the lifetime of the plan. The benefits are adjusted for 2.5% inflation.

<table>
<thead>
<tr>
<th>ROR</th>
<th>CWS Spending</th>
<th>Tax Rate</th>
<th>ORP Spending</th>
<th>Tax Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>59,700</td>
<td>13</td>
<td>60,000</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>64,100</td>
<td>12</td>
<td>66,000</td>
<td>16</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>69,100</td>
<td>15</td>
<td>72,000</td>
<td>17</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>74,900</td>
<td>16</td>
<td>78,000</td>
<td>17</td>
<td>4</td>
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<tr>
<td>8</td>
<td>80,100</td>
<td>16</td>
<td>85,000</td>
<td>16</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2: Social Security Scenario
The amount of money available for spending increases by $20,000 after taxes; ignoring Medicare Part B premiums. Social Security income reduces the efficiency impact of ORP but not the difference between the withdrawal rates (allowing for rounding error).

Figure 6 shows ORP’s withdrawal plan, for 5% investment return when a beginning $23,000 Social Security benefit is included.

Beyond adding Social Security benefits (purple line) and increasing spending (blue line) proportionally Figure 6 differs from Figure 1, the base scenario, in these aspects:

1. There is only a small initial transfer from IRA to Roth IRA. In the base scenario, Figure 1, the transfer to the Roth IRA filled out the 10% income tax bracket which would be empty. In the Social Security scenario the $1,000 transfer plus Social Security benefits filled out the 10% bracket (see Figure 7, later). Any additional transfers would be taxed at the 15% rate. ORP found it economically beneficial to keep funds in the IRA to take advantage of delayed taxing of the compounded IRA withdrawals until late in the plan. It is not automatic that funds are to be transferred to the Roth IRA even though there may be After-tax money available for spending.

2. The gap between the IRA and the Roth IRA withdrawals is smaller since the low IRA to Roth IRA transfers left the Roth IRA with a lower initial balance and the IRA with a higher balance at age 75, when the Roth IRA began distributions.

Figure 6 shows a more severe adjustment in the final years than Figure 1. The IRA jumped from $18,000 to $68,000 and the Roth IRA fell proportionally. The jump was caused by previously low IRA withdrawals and high Roth IRA withdrawals. Even so the last IRA withdrawal stayed inside the 15% tax bracket. This is evidence of ORP slowing withdrawals from the IRA until late in the plan to delay paying taxes.
As a side note this scenario was tried with an estate balance requirement added to the model. The effect was to eliminate the radical adjustment as the withdrawals from both accounts monotonically brought the accounts balance down to the desired estate balance. In other words the benefit from retaining funds in the IRA went into the estate rather than an end of plan adjustment.

4.1.3 Inflation Scenario

Table 3 compares ORP’s performance for the 5% ROR in higher inflation environments to that of CWS.

<table>
<thead>
<tr>
<th>Inflation</th>
<th>CWS</th>
<th>ORP</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>$41,400</td>
<td>$46,000</td>
<td>11%</td>
</tr>
<tr>
<td>3.5</td>
<td>36,200</td>
<td>41,000</td>
<td>6</td>
</tr>
<tr>
<td>5.0</td>
<td>29,300</td>
<td>33,000</td>
<td>3</td>
</tr>
<tr>
<td>7.1/5.8</td>
<td>24,200</td>
<td>29,000</td>
<td>5</td>
</tr>
<tr>
<td>13.5</td>
<td>5,000</td>
<td>8,000</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Effects of Inflation

The Age 95 column shows the inflated spending level at age 95.

All entries except 1 are for 5% investment returns. 5% inflation is the historical average for the United States, 1890-1995. The 7.1/5.8 represents the 1970's when inflation averaged 7.1% and stock returns were 5.8% [18]. Inflation in 1980 was 13.5%. [14].

Higher inflation lowers the amount of money available for spending in terms of today’s dollars as indicated by the Spending column. With higher inflation, the amount of money being withdrawn from savings increases more rapidly than in the lower inflation scenario. But the amount of money in retirement accounts does not change even though it now supports a higher spending level. To make the savings last the full term of retirement the starting value (at Age 65) has to be lower.

The highest tax rate is for the 2.5% scenario because it is initially withdrawing more money and the tax bracket limits are rising at a slower pace thanks to more stable inflation.

4.1.4 Illiquid Asset Scenario

Stocks and bonds are liquid assets. They can be sold at any time in any partial amount. A house or a dental practice is an illiquid asset. It can be sold one time only for the amount of its worth. Table 4 introduces an illiquid asset into the plan.
Tax Advantaged Account Withdrawal Strategies

In this example the illiquid asset is defined to be currently worth $400,000 with a cost basis of $100,000. The illiquid asset is not a home so that there is no capital gains exclusion. The asset value appreciates at the rate of inflation until the age of 85 when it is sold. The proceeds of the sale, less capital gains taxes, are transferred into the After-tax account and spent down from there.

The Efficiency column shown in Table 4 is similar to that shown in Table 1, here reproduced as Table 1 Efficiency.

Figure 8 shows the asset balances which include the illiquid asset that is sold at age 85.

---

<table>
<thead>
<tr>
<th>ROR</th>
<th>CWS Spending</th>
<th>Tax Rate</th>
<th>ORP Spending</th>
<th>Tax Rate</th>
<th>Efficiency</th>
<th>Table 1’s Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>$46,000</td>
<td>11%</td>
<td>$51,000</td>
<td>4%</td>
<td>11%</td>
<td>12%</td>
</tr>
<tr>
<td>5</td>
<td>50,900</td>
<td>11</td>
<td>56,000</td>
<td>7</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>54,600</td>
<td>12</td>
<td>61,000</td>
<td>6</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>60,400</td>
<td>13</td>
<td>67,000</td>
<td>8</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>64,800</td>
<td>14</td>
<td>75,000</td>
<td>7</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 4: Illiquid Asset Scenario

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Figure 8: Illiquid Asset Scenario, Asset Balances

The After-tax line (green) appears twice in Figure 8, once at the beginning and once at age 86 when the proceeds of the illiquid asset sale are transferred in. The illiquid asset line (purple) stops at age 85. The gap between the sale and the transfer of proceeds represents the capital gains tax. The IRA line (blue) is continuous across the plan. The Roth IRA account (red) is spent down to age 85 when the After-tax
account, replenished with the proceeds of the sale, takes over. At age 85 the total assets (light, top blue) has a kink when they are reduced by the amount of the capital gains tax paid on the sale of the illiquid asset.

4.1.5 Larger Beginning Account Balances Scenario

Table 5 compares ORP to CWS for different sizes of retirement savings accounts using the same assumptions as before, including 5% ROR.

<table>
<thead>
<tr>
<th>Balance</th>
<th>CWS Spending</th>
<th>CWS Tx Rate</th>
<th>ORP Spending</th>
<th>ORP Tx Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1M</td>
<td>$41,400</td>
<td>11%</td>
<td>$46,000</td>
<td>7%</td>
<td>11%</td>
</tr>
<tr>
<td>$2M</td>
<td>80,200</td>
<td>16</td>
<td>91,000</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>$3M</td>
<td>117,000</td>
<td>19</td>
<td>135,000</td>
<td>14</td>
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<td>$5M</td>
<td>191,500</td>
<td>22</td>
<td>220,000</td>
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<td>$10M</td>
<td>383,000</td>
<td>26</td>
<td>430,000</td>
<td>23</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 5: Plan Size Comparison

Efficiency values are in the same range regardless of the account size.

4.1.6 The IRA/Roth IRA Scenario

All of this has assumed significant account balances for all accounts at the start of retirement. Of the other combinations of non zero and zero balances the conventional wisdom will do just fine, with the exception of when the IRA balance is non zero. Table 6 shows the scenario when the IRA and Roth IRA contain beginning balances for different rates of investment return and the After-tax balance is zero.

<table>
<thead>
<tr>
<th>Rate</th>
<th>CWS Spending</th>
<th>CWS Tx Rate</th>
<th>ORP Spending</th>
<th>ORP Tx Rate</th>
<th>Efficiency</th>
<th>Table 1 Efficiency</th>
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</thead>
<tbody>
<tr>
<td>4%</td>
<td>$36,400</td>
<td>10%</td>
<td>$40,000</td>
<td>8%</td>
<td>10</td>
<td>12%</td>
</tr>
<tr>
<td>5</td>
<td>41,000</td>
<td>10</td>
<td>46,000</td>
<td>9</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>45,800</td>
<td>11</td>
<td>52,000</td>
<td>8</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>51,500</td>
<td>11</td>
<td>58,000</td>
<td>9</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>57,500</td>
<td>13</td>
<td>65,000</td>
<td>10</td>
<td>13</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 6: IRA/Roth IRA Scenario

In this case the $1,000,000 retirement savings are split between the IRA ($546,000) and Roth IRA ($454,000) accounts. The initial balances were computed using the same technique as was used in the three account scenario.

For comparison purposed the Table 1 Efficiency column shows the efficiency from Table 1, the base scenario.

Figure 9 shows the withdrawal patterns for this scenario.
The interesting features of Figure 9 are before the age of 70, when the RMD begins, and after the age of 86 when the RMD is in decline. During both periods Roth IRA withdrawals exceed IRA withdrawals. There are IRA withdrawals in every year to keep each year’s IRA withdrawals in lower tax brackets. Figure 9 indicates that IRA withdrawals are lower so as to delay some taxable withdrawals as long as possible to pay taxes as late as possible. After the age of 92 IRA withdrawals increase dramatically, but stay in the 10% tax-bracket, as it is liquidated.

It may be noted that in the absence of other income, the IRA and Roth IRA lines are the mirror image of each other. This is because IRA withdrawals are either trying to minimize taxes or being constrained by the RMD. The Roth IRA is filling in the difference to support the spending plan.

This pattern remains the same when initial Roth IRA balances that are equal to or greater than IRA balances. Redoing this scenario with an estate requirement resulted in behavior similar to that discussed earlier in the Social Security scenario.

In other word, the more economical strategy is to delay some of the IRA withdrawals and accelerate some of the Roth IRA withdrawals. This observation is contrary to the conventional wisdom that IRA withdrawals should take precedence over Roth IRA withdrawals.

4.1.7 The IRA/After-tax Account Scenario

Table 7 shows the results of when there is no Roth IRA but there are significant beginning IRA ($580,000) and After-tax ($420,000) balances.
The proportions for the initial balances were computed using the same technique as for the three account, base scenario. Comparing the Efficiency column of Table 7 to Table 1, the base scenario, shows that having money in all three accounts offers some advantage over having no initial money in the Roth IRA.

Figure 10 shows the cash flow for this scenario across retirement.

Figure 10 exhibits an unfortunate feature of LP: the alternate optimal, which gives a solution that is optimal but not practical to implement. Section 3 stated that there is no better solution than the one reported by ORP, but alas it may not be unique. In this case, early in the plan there are 2 spikes of distributions from the IRA for transfers to the Roth IRA. The spikes fund the Roth IRA to the point that it can take over for the After-tax when it becomes depleted at age 83.
The rest of the plan is pretty much as to be expected. When the After-tax account is depleted the Roth IRA takes up the slack at pretty much the same level. While the Roth IRA is being distributed at a higher than expected the IRA is following the RMD down as the IRA is being depleted. Some of the IRA money is being retained to delay paying income taxes as long as possible, which accounts for the withdrawal adjustments at the end.

Figure 11 shows how IRA withdrawals are falling into the Federal income tax brackets.

![Figure 11: Tax Brackets for the IRA/After-tax Scenario](image)

The early spikes stay within the 15% tax bracket, the same as when withdrawals are being forced by the RMD. There does not seem to be any reason for the IRA withdrawal spikes. It is interesting to note that the withdrawals late in the plan are at the top of the no tax bracket.

### 4.1.8 IRA Only Scenario

Even when the IRA is the only account containing money ORP still has to room to maneuver. The strategy is to move money into the other accounts early on and then use them to supplement IRA withdrawals later. The IRA Only Scenario begins with $1,000,000 in the IRA and zero balances in the other two accounts. Table 8 demonstrates:
Table 8: IRA Only Scenario

<table>
<thead>
<tr>
<th>Age</th>
<th>IRA</th>
<th>After-tax</th>
<th>Roth IRA</th>
<th>IRA2Roth</th>
<th>Savings</th>
<th>Taxes</th>
<th>Spending</th>
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</thead>
<tbody>
<tr>
<td>65</td>
<td>58</td>
<td>9</td>
<td>7</td>
<td>42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>59</td>
<td>9</td>
<td>7</td>
<td>43</td>
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</table>

Table 9: Single Account Scenario Withdrawal Report ($000)

Initially there is no Roth IRA nor After-tax accounts so ORP invents them.

IRA to Roth IRA transfers (column IRA2Roth) begin at age 65. At age 70, when the RMD begins, IRA to Roth IRA transfers decrease because the RMD cannot be used to fund transfers to the Roth IRA. Instead the funds, which would have been sent to the Roth IRA, are transferred to the After-tax account (column Savings). At age 79 IRA withdrawals (column IRA) are reduced to the level of the RMD so that no additional transfers can be made to the Roth IRA and transfers to the After-Tax account end the
following year. The year after that distributions tied to the RMD decline as the IRA balance declines and the After-tax account (column After-tax) supplements IRA distributions to meet spending requirements.

At age 87 the After-tax account is depleted and withdrawals from the Roth IRA begin (column Roth IRA). In the final year, Age 95, the Roth IRA is depleted and one large, last withdrawal is made from the IRA to fund that year’s spending requirements and leave the estate at zero.

In order to level out taxes paid over time, money taken from the IRA before it is needed has to stored somewhere:

1. Roth IRA
2. After-tax account
3. The IRA itself, by keeping a limited amount in the IRA until the very end.

Table 9 is an excellent example of using all three storage places.

All IRA distributions stay inside the 15% tax bracket even the last big one.

### 4.1.9 Account Balance Scenarios

Most of the scenarios up to this point assumed beginning account balances are more or less of the same magnitude and that the IRA is the largest. Table 10 explores a range of IRA and Roth IRA beginning account balances. The scenarios are labeled according to the percentage of $1,000,000 that is in the account at the beginning. No allowance is made for taxes on the Roth IRA contributions. There is no After-tax initial balance.

<table>
<thead>
<tr>
<th>IRA</th>
<th>Roth IRA</th>
<th>CWS Spending</th>
<th>ORP Spending</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>90%</td>
<td>$44,300</td>
<td>$47,000</td>
<td>6%</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
<td>43,900</td>
<td>47,000</td>
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<td>30</td>
<td>70</td>
<td>41,800</td>
<td>47,000</td>
<td>12</td>
</tr>
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<td>41,700</td>
<td>47,000</td>
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<tr>
<td>90</td>
<td>10</td>
<td>40,200</td>
<td>43,000</td>
<td>7</td>
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</tbody>
</table>

**Table 10: Extreme Balance Scenarios**

The penultimate table entry (90/10) is probably the most typical of most retirees. The 60/64 row is similar to the IRA/Roth IRA scenario.

Figure 12 is a chart of the IRA distributions for the 10 scenarios in Table 10.
Figure 12: IRA Distributions for Thee Scenarios in Table 10.

The legend labels the lines according to the percentage of the funds in the IRA for each scenario; the remainder are in the Roth IRA. For example the blue line at the bottom represents 10% IRA and 90% Roth IRA accounts.

All of the distribution lines possess the familiar pattern of the earlier scenarios. Before the RMD, withdrawals are made from the IRA for transfers to the Roth IRA except for the 10% (blue) line where no IRA distributions are made until the RMD begins. When the RMD starts the IRA distribution pegs to the RMD level until age 92 or so. Then the IRA distribution spikes when the funds being retained in the IRA for tax reasons are liquidated to bring the account balance to zero. Rather than take the last withdrawal into a higher tax bracket ORP will make oversized withdrawals for two or three years before the end of the plan. If, at age 92, the retiree discovers herself to be in good health she will have a cushion to for over the next few years thereby averting one of the retirement risks – outliving the retirement plan.

4.1.10 Summary

When all three accounts have the same ROR ORP will spread withdrawals from the IRA across the entire term to achieve lower taxes. ORP will try to push annual IRA distributions down into a low tax bracket while at the same time satisfying the RMD and maximizing total distributions by delaying IRA distributions as long as possible. Figure 2 pointed to the RMD as the important factor in setting IRA distribution levels. Except for the last few years IRA withdrawals are pegged to the RMD when it is active. The RMD and thus IRA withdrawals will be in a narrow range. Even when Social Security benefits are added to the mix in Figure 5 the IRA distributions are tied to the RMD.
4.2 Multiple Rates of Investment Return

Asset diversification will have a different class of asset with different RORs invested in each account. Table 11 shows a set of scenarios where each account has its own ROR.

<table>
<thead>
<tr>
<th>ROR</th>
<th>CWS Spending</th>
<th>CWS Tx Rate</th>
<th>ORP Spending</th>
<th>ORP Tx Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/5/5</td>
<td>$42,600</td>
<td>11%</td>
<td>$46,000</td>
<td>7%</td>
<td>8%</td>
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<td>7/5/3</td>
<td>45,200</td>
<td>11</td>
<td>51,000</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>7/3/5</td>
<td>40,600</td>
<td>10</td>
<td>51,000</td>
<td>9</td>
<td>26</td>
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<tr>
<td>5/7/3</td>
<td>50,000</td>
<td>11</td>
<td>55,000</td>
<td>0</td>
<td>10</td>
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<tr>
<td>5/3/7</td>
<td>37,000</td>
<td>10</td>
<td>49,000</td>
<td>0</td>
<td>32</td>
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<tr>
<td>3/7/5</td>
<td>47,400</td>
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<tr>
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<td>39,300</td>
<td>10</td>
<td>49,000</td>
<td>0</td>
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</table>

Table 11: Investment Return Scenarios

The ROR column shows how the returns are assigned to the different accounts; IRA/Roth IRA/After-tax. For example the last row has the IRA returning 3%, the Roth IRA 5% and the After-Tax 7%. The first row is for the base scenario of Table 1 which has all three accounts with 5% RORs. The average ROR for the entries in Table 11 is 5%. Having three different ROR’s assigned to the three different accounts gives significantly improved spending as compared to the base case. This is because ORP has the freedom to exploit the differences of the RORs within the IRA tax situation.

The two scenarios in lines 2 and 3 assign the highest ROR to the IRA. The pattern of the withdrawal report is congruent with the base scenario although the magnitudes are larger thanks to a higher ROR on the largest account.

In the remaining scenarios accounts other than the IRA have the highest ROR and this changes things. In these scenarios no taxes are paid at age 75 because large IRA transfers to other were completed before age 75. These IRA withdrawals are all in the 15% tax bracket. By the age of 75 the IRA is depleted and no further taxes are paid.

A typical scenario is the one with the highest spending is 5/7/3, i.e. rates of return of 5% for the IRA, 7% for the Roth IRA and 3% for the After-tax account. Figure 11 shows the Withdrawal Report for the 5/7/3 scenario;
The After-tax account, with the lowest rate of return, is distributed early for spending. Since the IRA is returning 5% and the Roth IRA 7% there is an economic incentive to transfer the IRA to the Roth IRA all in the 15% tax bracket. This is done from age 65 to 69, while the After-tax account is being spent. From the ages of 70 to 79 the IRA and Roth IRA are distributed in parallel to satisfy spending. At age 80 the IRA is depleted, partly because of spending and partly because of transfers. Spending is satisfied with Roth IRA alone, with no taxes paid. ORP will pay taxes on the early transfer to get a better ROR later on.

Having different RORs for the different accounts puts a new level of complexity into the model. Now not only is ORP trying to minimize taxes but it is also trying to maximize returns by moving money from the IRA to higher earning accounts. Restrictions in the tax code and in the model only allow for transfers from the IRA to the Roth IRA and After-tax accounts.

Figure 12 presents the withdrawal results for the 3/5/7 scenario where the largest account has the smallest ROR and the smallest account has the largest ROR.
ORP can’t empty the IRA fast enough. The large IRA withdrawal at age 65 is in the 25% bracket. Ages 66 to 71 are in the 15% bracket and from age 72 and after they are too low to be taxed. Most of the IRA withdrawals are transferred into the After-tax account with some used for spending. The Roth IRA is used for spending until it is depleted at age 73. After-tax withdrawals do not start until age 73 since this account is earning the superior return. The further away from age 65, the lower the tax bracket on IRA withdrawals.

Another scenario is to eliminate the After-tax account initial balance and make it a two account scenario. The pattern is exactly the same with the Roth IRA supporting spending while the IRA is rolled over into the After-tax account. After-tax account distributions do not begin until the Roth IRA is depleted.

The thrust of this is that when the IRA has a lower ROR it is rolled over into the account with the highest ROR as quickly as possible while keeping the tax burden down. At age 65 ORP calculates that paying the extra tax on rollovers to the After-tax account will payed back with the higher ROR.

5 Conclusion

Some conclusions which can be drawn are:
1. Optimized withdrawal strategies from tax-advantaged accounts are superior to the conventional wisdom.
2. Withdrawing from retirement accounts in parallel, rather than serially, increases the amount of money available from tax advantaged accounts for spending during retirement.
DRAFT
Tax Advantaged Account Withdrawal Strategies
J. Welch  September 20, 2011

3. In many cases before the age of 90, IRA withdrawals are constrained by the RMD.
4. Tax-efficiency causes part of the withdrawals from the IRA to be delayed to the end of the term. IRA distributions may be lower and Roth IRA distributions higher than intuitive expectations.
5. Inflation is death to retirement spending.
6. Social Security benefits are important to withdrawal strategies because of their tax consequences and they will affect withdrawal patterns.
7. Efficient withdrawal strategies are just as important to large retirement savings accounts as they are to modest accounts.
8. When there are no other sources of income, IRA and Roth IRA distributions are mirror images of each other.
9. Diversification pays; when accounts have different rates of return the optimal plan may provide more spending than a plan computed for all accounts having the same ROR.
10. If the IRA ROR is significantly less than either of the other accounts then the efficient strategy is to make large transfers from the IRA to one of the other accounts at the start of retirement to take advantage of the higher ROR.
11. Some of the things that ORP does may seem arbitrary at first glance but upon examination they make economic sense.

In his paper about optimizing IRA and Roth IRA withdrawals under the progressive Federal Income tax structure Horan [10] arrives at the following withdrawal strategy: “… taking traditional IRA distributions up to the top of a ‘low’ tax bracket and satisfying the remainder of the withdrawal requirement from the Roth IRA yields residual accumulations that are substantially greater than the [conventional wisdom] strategy”

In their paper about maximizing Social Security benefits by reducing personal income taxes Mahaney and Carlson [4] propose an IRA distribution strategy similar to Horan’s.

ORP’s results indicate that there are two dramatically different situations. If the IRA ROR is not less than that of the Roth IRA then ORP partially confirms Horan’s strategy. Under this condition before age 70, before the RMD is inactive, and after age 90, when the RMD is no longer a factor, ORP follows Horan’s advice and limits withdrawals to the upper bound of the lowest possible tax bracket. In between the IRA withdrawals are constrained from below by the RMD whatever the tax bracket.

Horan goes on to recommend these “avenues for future research”:

1. “Mandatory distribution requirements from traditional IRAs.
3. “Integrate the impact on withdrawal location of taxable accounts outside traditional IRA and Roth IRA plans” (e.g. after-tax accounts, illiquid assets).

All of these areas have been explored in this paper.

ORP is not the only application of linear programming to optimizing withdrawals from tax-advantaged retirement accounts. Two others are Ragsdale [17] in 1993 and Coopersmith [19] in 2011.

ORP is unique for other reasons:
• ORP maximizes spending rather than maximizing the estate, which is more meaningful to the individual user.
• ORP is a classic Operations Research model and system [23] running on a web server. It is probably the only linear programming application available for use by the general public over the Internet.
• ORP was built using off-the-shelf commercial components rather than being a spreadsheet application on a desk top computer. (ORP’s matrix description language was implemented for the petroleum process industry [20] and its optimizer [21] was originally used for agricultural feed blending.)
• ORP models all three retirement account types, the RMD, the Federal progressive income tax and a variety of supplementary income streams.
• ORP models rollovers from the IRA to the Roth IRA and After-tax accounts.
• ORP models both the savings accumulation and distribution phases of retirement planning.
• ORP produces answers in a few seconds except when the server is heavily loaded.
• ORP is useful as a retirement planning tool for individual use because its parameter set can be tailored to an individual’s particular situation.
• ORP is available to everyone at no charge on the Internet: www.i-orp.com

ORP will produce a wide variety of results for different circumstances. Factors such as pensions, Social Security benefits, selling illiquid assets, reverse mortgages and employment earnings during retirement will change ORP’s results, sometimes dramatically.

Trying to fit an individual’s situation into a set of published guidelines is inherently difficult and inefficient. It is more meaningful just to run the model.
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