Abstract

The two phases of retirement planning are 1) pre retirement saving (accumulation) and 2) retirement spending (distribution). This paper describes a technique for maximizing the amount of money available for spending during the distribution phase.

1. The objective of a retirement plan is to enjoy constant annual income for spending throughout retirement (no plan failure) while leaving a modest estate (no large unspent surplus).
2. Spending sources of funds are Social Security benefits, pension(s) and retirement savings.
3. Retirement savings may be in three different account types; 1) tax-deferred, 2) Roth IRA and 3) after-tax. Tax-deferred withdrawals are subject to progressive, personal income taxes. Withdrawals from the others are untaxed.
4. The scheduling of withdrawals from retirement savings will determine how much money is available for spending after paying taxes on tax-deferred withdrawals as well as other income sources while meeting the IRS imposed Required Minimum Distribution (RMD) on the tax-deferred account.
5. The scheduling of withdrawals can be represented as a Linear Programming (LP) model. LP is an Operations Research tool that has been in use since the 1950’s.
6. Solving the LP model will produce a plan that satisfies the retirement plan’s objectives of Item 1 above.

Conventional wisdom, as advocated by many financial advisers and the financial press, calls for retirement savings to be sequentially liquidated in the order: after-tax account, tax-deferred account and finally, the Roth IRA account.

This paper reports how much the LP model improves on conventional wisdom as measured by each plan’s annual spending. For most scenarios LP outperforms conventional wisdom by 10% or more. Even for the scenario where only the tax-deferred account has a beginning balance the LP model can improve on the conventional wisdom.

This paper extends previous work in the area of tax efficient withdrawal strategies by including IRA required minimum distributions, progressive Federal income tax tables, after-tax accounts and other sources of taxable retirement income into the model. The LP model includes provision for partial tax-deferred account transfers to the other accounts.
1 Introduction

“Job one has to be to protect [retirement savings] from taxes because that's going to be a bigger hit than any investment”. [26]

Ed Slott, CPA; Named as “The Best” source for IRA advice by The Wall Street Journal

This paper addresses the question of: “How much does Linear Programming (LP) improve over the conventional wisdom for withdrawing funds from retirement savings accounts?”

LP is an Operations Research tool that has been a successful computer application since the late 1950’s [22]. One useful LP application is to maximize the funds available for spending from annual retirement saving account withdrawals combined with other retirement income.

The essential element of the optimal plan is that when the tax situation is favorable money is withdrawn from the tax-deferred account and transferred to other types of accounts and later those accounts supplement tax-deferred withdrawals to meet spending requirements.

2 Background

The three different types of retirement savings accounts are:

1. **Tax-deferred (IRA):** There are no income taxes on employment earnings contributed to the IRA but all withdrawals are taxed as personal income. This type of account includes IRA, 401k, 403b and a variety of others, all of which are generically the same. The term IRA will be used herein to denote tax-deferred accounts since most of the others are rolled over into an IRA before or at retirement. IRAs have a **Required Minimum Distribution (RMD).** The RMD is an amount that the IRS requires be withdrawn annually beginning at the age of 70½. It is computed as the IRA account balance on January 1 divided by a life expectancy value taken from an IRS published table. [15].

2. **Roth IRA:** Income taxes are paid on the employment income contributions to a Roth IRA but there are no taxes on withdrawals. In addition to employment contributions withdrawals from an IRA may be transferred into a Roth IRA after personal income taxes have been paid on the withdrawals.

3. **After-tax account:** Contributions to the After-tax account can be from any source, after taxes have been paid. Taxes are paid on sales, dividends and interest annually. No taxes are paid on
withdrawals. IRA withdrawals may be transferred into the After-tax account after taxes have been paid.

From the perspective of how they are taxed, these accounts are three entirely different entities.

After the retiree reaches the age of 59 1/2 withdrawals can be made from any account in any amount without penalty. The amount of withdrawals and the order in which they are made (the withdrawal plan) will affect the amount of money available annually, after taxes, for spending over the term of retirement. The spending for different withdrawal plans will vary because of the Federal progressive income tax and the RMD.

The goal of a withdrawal plan is to maximize spending. The relative efficiency of one plan over another is the percentage improvement of the plan with the larger spending to the smaller.

The conventional wisdom offered by many financial advisers [25] and recommended by the financial press is:

1. draw down the After-tax account first,
2. draw down the IRA second,
3. draw down the Roth IRA last.

Raabe and Toolson [7] showed that the conventional wisdom is more efficient than any other permutation of sequential account distribution strategies.

The conventional wisdom is not tax-efficient because it pays no income taxes either during the first distribution phase when the After-tax account is being drawn down or during the third phase when the Roth IRA is being drawn down. This compresses IRA taxable withdrawals into the middle phase which will likely push part of these distributions into a higher income tax bracket than if they were spread over the entire retirement term.

3 The Experiment

Two computer programs were used to compare the efficiency of a LP optimized plan to the conventional wisdom:

1. The Optimal Retirement Planner (ORP) [8] is the LP application that was used to compute the optimal plans. ORP accepts a model consisting of a set of activities that can be done and constraints on those activities. It then computes a solution to the model that has some maximum economic advantage [6]. In this case ORP maximizes spending for each year of retirement, after income taxes are paid. LP mathematically guarantees that there is no better solution than the one computed [16].
2. The Conventional Wisdom Simulator (CWS) is a spreadsheet that simulates the conventional wisdom over the term of retirement. CWS was implemented for the purpose of providing a benchmark to measure the efficiency of ORP generated plans.

The two programs use the same parameter set and compute to the same objective; maximum spending over retirement. Spending for age 65 is in today’s dollars and spending for subsequent years is subjected to compounded inflation. The value of a computed plan is measured by spending at age 65.

Given a set of parameters the program’s objective is to find the highest spending level that will not deplete retirement funds before the end of the plan (no plan failure) and will leave a zero balance in the estate (no unspent surplus). Both programs annually reduce the IRA balance by the IRA withdrawal plus taxes paid. This means that the IRA balance goes down by more than just the withdrawal for spending. Taxes were computed using 2011 IRS tables. For a given set of parameters, spending computed by the two programs can be compared and the more efficient plan can be judged.

The situation being modeled is a 65 year old, unmarried retiree who saved $1,000,000 for retirement. The IRA contains $400,000, the Roth IRA $350,000 and the After-tax account $250,000. These proportions were arrived at by running ORP’s accumulation phase for a 30 year old allocating 1/3 of her total retirement savings to each of the three accounts. Contributions were reduced by income taxes paid for the Roth IRA and After-tax account contributions. The plan was evaluated at age 65. The Roth IRA account balance at age 65 is lower than the IRA because of income taxes paid on the Roth IRA contributions. The After-tax amount is lower yet because not only were income taxes paid on contributions but 15% capital gains taxes were paid on annual returns. The rate of return (ROR) during the accumulation phase was 5% for all three accounts.

Other assumptions are, unless specific for a given scenario, a 2.5% inflation rate and a 5% investment ROR.

4 Computational Results

The experiment was to run the two programs for the parameter set described above and to compare the efficiency results. Then the ORP detail results are examined to understand the dynamics of the optimal plan. This process is repeated for additional scenarios created by changing parameters or including an additional feature in the base scenario.

4.1 Single ROR

The simplest scenarios to explore are those in which all accounts have the same ROR. This allows ORP to clearly demonstrate the consequences of personal income tax minimization. Later, in section 4.2, different ROR’s for different accounts will introduce more complex economics.
4.1.1 Base Scenario

The Base Scenario is for the parameter set described above. Table 1 reports spending in today’s dollars, computed by CWS and ORP, for different RORs.

<table>
<thead>
<tr>
<th>ROR</th>
<th>Spending (CWS)</th>
<th>Tax Rate</th>
<th>Spending (ORP)</th>
<th>Tax Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>$36,600</td>
<td>10%</td>
<td>$41,000</td>
<td>7%</td>
<td>12%</td>
</tr>
<tr>
<td>5</td>
<td>42,600</td>
<td>11%</td>
<td>46,000</td>
<td>7%</td>
<td>8%</td>
</tr>
<tr>
<td>6</td>
<td>46,000</td>
<td>11%</td>
<td>53,000</td>
<td>6%</td>
<td>15%</td>
</tr>
<tr>
<td>7</td>
<td>53,200</td>
<td>12%</td>
<td>59,000</td>
<td>8%</td>
<td>11%</td>
</tr>
<tr>
<td>8</td>
<td>57,700</td>
<td>13%</td>
<td>66,000</td>
<td>7%</td>
<td>14%</td>
</tr>
</tbody>
</table>

Table 1: Comparison of CWS to ORP

ROR is the rate of investment return; the parameter that was varied for the results in this table. The Spending columns show the plan’s spending for the first year of retirement in today’s dollars for the different ROR values. The Tax Rate columns contain the effective personal income tax rate on IRA withdrawals. It is computed as taxes paid on the IRA withdrawal divided by the IRA withdrawal for age 75. Age 75 was judged to be more or less representative of both plans’ withdrawals. The Efficiency column measures the advantage of the ORP withdrawal plan over CWS with the spending difference as a percentage of CWS spending at age 65:

\[
\text{Efficiency} = \frac{\text{ORP spending} - \text{CWS spending}}{\text{CWS spending}}.
\]

Figure 1 demonstrates the dynamics of the ORP withdrawal plan for the 5% ROR. It shows how money is withdrawn from the three retirement accounts over the term of retirement.
Withdrawals from the IRA (blue) stay in a relative narrow band throughout retirement. This keeps income taxes in a low tax bracket.

Until age 70 the spending line (purple) and the After-tax withdrawal line (red) are the same (top line) because spending is being funded exclusively from the After-tax account. IRA withdrawals during this period are rolled over into the Roth IRA account. At age 70 the RMD kicks in, increasing the required withdrawals from the IRA and correspondingly lowering the withdrawals from the After-tax account. RMD money must be spent because, by law, it cannot be rolled over into a Roth IRA. At age 71 the After-tax account is depleted and withdrawals begin from the Roth IRA (green). At age 71 spending is partially funded by all three accounts. After age 71 the After-tax account is depleted and parallel withdrawals from the IRA and Roth IRA fund spending. At age 91 IRA withdrawals begin to rise while the Roth IRA begins to tail off. At age 95 both accounts are taken to zero. The ORP strategy of delaying some IRA withdrawals until the end of the plan is apparent in the large withdrawal in the final year which is offset by a corresponding reduced Roth IRA withdrawal to meet annual spending.

Figure 2 shows IRA account activity during retirement and gives a perspective as to what is driving the IRA distributions.
The blue line represents IRA distributions. The red line, to the left, represents IRA to Roth IRA transfers. The RMD level (green) is fixed by law according to the balance of the IRA and not subject to optimizer modification. After age 70 IRA distributions are being constrained by the RMD until age 91. The IRA distributions are seeking a lower level but it cannot go below the RMD. Because of the way it is defined the RMD remains remarkably flat for most of retirement. Looking back at Figure 1 it is seen that as the RMD, thus IRA withdrawals, decline the Roth IRA is filling in to meet spending requirements. At age 91 spending needs are large enough such that the RMD no longer constrains IRA withdrawals.

During most of the plan ORP would prefer to distribute less from the IRA and more from the Roth IRA. Were it not for the RMD the blue line would continue at the initial level of the red line well into retirement. Clearly the RMD is costing retirees money although how much is yet to be explored.

It might be the case that since the IRA requires that the RMD be calculated using a life expectancy in the 90's that shortening the retiree's specified life expectancy to 85 would raise the IRA withdrawals above the RMD. Experiments showed that the IRA withdrawals stays fixed to the RMD.

Figure 3 shows how IRA withdrawals fit into the Federal progressive income tax brackets.
Each vertical bar represents IRA withdrawals for one age. For example, the age 65 bar shows $20,000 worth of withdrawals, divided into the No Tax and 10% brackets. The No Tax bracket excludes $11,000 of withdrawals from taxation because of the standard deduction and one personal exemption of the progressive income tax.

Before age 70 IRA withdrawals are at the upper bound of the 10% bracket but no more. After the RMD starts up withdrawals are forced into the 15% bracket. Looking back at Figure 1 and Figure 2, the IRA withdrawals stay with the RMD all the way to age 91. There all IRA withdrawals are in the No Tax bracket until the final year when the final large IRA withdrawal returns to the 10% bracket.

Figure 4 shows the annual asset balances of the three accounts which reflect the withdrawals shown in Figure 1.
The After-tax account decreases to zero from the beginning of retirement to age 73 while it is being drawn down for spending. Before the age of 70 the IRA increases in value since transfers to the Roth IRA are smaller than its asset returns (5%).

At age 75, when Roth IRA withdrawals begin, the balance of the Roth IRA is significantly larger than the IRA because the IRA started distributions 5 years earlier and because of the transfers from the IRA to the Roth IRA.

Figure 5 shows the account balances computed by CWS. It compares to the ORP account balances shown in Figure 4.
CWS’s strategy it to spend down the retirement accounts sequently. From ages 65 to 70 the After-tax account is used for spending and the IRA and Roth IRA accounts compound in an identical fashion. From age 70 to 75 the RMD requires IRA distributions even though there is still After-tax money available for spending. After-tax withdrawals proceed at a reduced rate because the RMD withdrawals from the IRA are now a part of spending. When the After-tax account is depleted spending is only from the IRA while the Roth IRA continues to appreciate. After the IRA is depleted then spending is funded by the Roth IRA to the end of the plan.

4.1.2 Social Security Benefits Scenario

Table 2 shows the base scenario with $23,000 of annual Social Security benefits added to the retirement plan. This scenario demonstrates the impact of a steady stream of income over the lifetime of the plan. The Social Security benefits are adjusted for 2.5% inflation.

<table>
<thead>
<tr>
<th>ROR</th>
<th>Spending</th>
<th>Tax Rate</th>
<th>CWS</th>
<th>Spending</th>
<th>Tax Rate</th>
<th>ORP</th>
<th>Tax Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>57,700</td>
<td>13</td>
<td>60,000</td>
<td>10%</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>62,000</td>
<td>12</td>
<td>66,000</td>
<td>9</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>66,700</td>
<td>15</td>
<td>72,000</td>
<td>10</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>72,300</td>
<td>16</td>
<td>78,000</td>
<td>11</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>77,300</td>
<td>16</td>
<td>85,000</td>
<td>10</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Social Security Scenario

The amount of money available for spending increases by $20,000 after taxes; ignoring Medicare Part B premiums. The Efficiency column is computed with $20,000 of Social Security benefits deducted from the Spending columns.
Beyond adding Social Security benefits (purple line) and increasing spending (blue line) proportionally Figure 6 differs from Figure 1, the base scenario, in these aspects:

1. There is only a small initial transfer from IRA to Roth IRA. In the base scenario, Figure 1, the transfer to the Roth IRA filled out the 10% income tax bracket. In the Social Security scenario the $1,000 transfer plus Social Security benefits filled out the 10% bracket. Any additional transfers would be taxed at the 15% rate. ORP found it economically beneficial to keep funds in the IRA to take advantage of delayed taxing of the compounded IRA withdrawals until late in the plan. It is not automatic that funds are to be transferred to the Roth IRA even though there may be After-tax money available for spending.

2. The gap between the IRA and the Roth IRA withdrawals is smaller since the low IRA to Roth IRA transfers left the Roth IRA with a lower initial balance and the IRA with a higher balance at age 75, when the Roth IRA began distributions.

Figure 6 shows a more severe adjustment in the final years than Figure 1. The IRA jumped from $18,000 to $68,000 and the Roth IRA fell proportionally. The jump was caused by previously low IRA withdrawals and high Roth IRA withdrawals. Even so the last IRA withdrawal stayed inside the 15% tax bracket. This is evidence of ORP slowing withdrawals from the IRA until late in the plan to delay paying taxes.

As a side note this scenario was tried with a non zero estate requirement. The effect was to eliminate the radical adjustment as the withdrawals from both accounts monotonically brought the accounts balance down to the desired estate balance. In other words the benefit from retaining funds in the IRA went into the estate rather than an end of plan adjustment.
4.1.3 Inflation Scenario

Table 3 compares ORP’s performance for the 5% ROR in higher inflation environments to that of CWS.

<table>
<thead>
<tr>
<th>Inflation</th>
<th>CWS Spending Age 95</th>
<th>CWS Tax Rate</th>
<th>ORP Spending Age 95</th>
<th>ORP Tax Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>$41,400</td>
<td>11%</td>
<td>$46,000</td>
<td>7%</td>
<td>11%</td>
</tr>
<tr>
<td>3.5</td>
<td>36,200</td>
<td>10</td>
<td>41,000</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>5.0</td>
<td>29,300</td>
<td>9</td>
<td>33,000</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>7.1/5.8</td>
<td>24,200</td>
<td>6</td>
<td>29,000</td>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>13.5</td>
<td>5,000</td>
<td>0</td>
<td>8,000</td>
<td>0</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 3: Effects of Inflation

The Age 95 columns show the inflated spending level at age 95.

All entries except one are for 5% investment returns. 5% inflation is the historical average for the United States, 1890-1995. The 7.1/5.8 represents the 1970’s when inflation averaged 7.1% and stock returns were 5.8% [18]. Inflation in 1980 was 13.5%. [14].

Higher inflation lowers the amount of money available for spending in terms of today’s dollars as indicated by the Spending columns. With higher inflation, the amount of money being withdrawn from savings increases more rapidly than in the lower inflation scenario. But the amount of money in retirement accounts does not change even though it now supports a higher spending level. To make the savings last the full term of retirement the starting value (at Age 65) has to be lower.

The highest tax rate is for the 2.5% scenario because it is initially withdrawing more money and the tax bracket limits are rising at a slower pace thanks to more stable inflation. Inflation is a tax that cannot be scheduled.

4.1.4 Illiquid Asset Scenario

Stocks and bonds are liquid assets. They can be sold at any time in any partial amount. A house or a dental practice is an illiquid asset. It can be sold one time only for the amount of its worth. Table 4 introduces an illiquid asset into the plan.
In these scenarios the illiquid asset is defined to be currently worth $400,000 with a cost basis of $100,000. The illiquid asset is not a home so that there is no capital gains exclusion. The asset value appreciates at the rate of inflation until the age of 85 when it is sold. The proceeds of the sale, less capital gains taxes, are transferred into the After-tax account and spent down from there.

The Efficiency column shown in Table 4 is similar to that shown in Table 1, here reproduced as Table 1’s Efficiency.

Figure 8 shows the asset balances which include the illiquid asset that is sold at age 85.
account, replenished with the proceeds of the sale, takes over. At age 85 the total assets (turquoise, across the top) has a kink when they are reduced by the amount of the capital gains tax paid on the sale of the illiquid asset.

4.1.5 Larger Beginning Account Balances Scenario

Table 5 compares ORP to CWS for different sizes of retirement savings accounts using the same assumptions as before, including 5% ROR.

<table>
<thead>
<tr>
<th>Balance</th>
<th>CWS Spending</th>
<th>CWS Tx Rate</th>
<th>ORP Spending</th>
<th>ORP Tx Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1M</td>
<td>$41,400</td>
<td>11%</td>
<td>$46,000</td>
<td>7%</td>
<td>11%</td>
</tr>
<tr>
<td>$2M</td>
<td>80,200</td>
<td>16</td>
<td>91,000</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>$3M</td>
<td>117,000</td>
<td>19</td>
<td>135,000</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>$5M</td>
<td>191,500</td>
<td>22</td>
<td>220,000</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>$10M</td>
<td>383,000</td>
<td>26</td>
<td>430,000</td>
<td>23</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 5: Plan Size Comparison

Efficiency values are in the same range regardless of the account size.

4.1.6 The IRA/Roth IRA Scenario

All of this has assumed significant account balances for all accounts at the start of retirement. Of the other combinations of non zero and zero balances the conventional wisdom will do just fine, with the exception of when the IRA balance is non zero.

Table 6 shows the scenario when the IRA and Roth IRA contain beginning balances for different rates of investment return and the After-tax balance is zero.

<table>
<thead>
<tr>
<th>ROR</th>
<th>CWS Spending</th>
<th>CWS Tx Rate</th>
<th>ORP Spending</th>
<th>ORP Tx Rate</th>
<th>Efficiency</th>
<th>Table 1 Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>$36,400</td>
<td>10%</td>
<td>$40,000</td>
<td>8%</td>
<td>10</td>
<td>12%</td>
</tr>
<tr>
<td>5</td>
<td>41,000</td>
<td>10</td>
<td>46,000</td>
<td>9</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>45,800</td>
<td>11</td>
<td>52,000</td>
<td>8</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>51,500</td>
<td>11</td>
<td>58,000</td>
<td>9</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>57,500</td>
<td>13</td>
<td>65,000</td>
<td>10</td>
<td>13</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 6: IRA/Roth IRA Scenario

In this case the $1,000,000 retirement savings are split between the IRA ($546,000) and Roth IRA ($454,000) accounts. The initial balances were computed using the same technique as was used in the three account scenario. For comparison purposes the Table 1 Efficiency column shows the efficiency from Table 1, the base scenario.

A conclusion is that an After-tax balance is not a requirement for making good use of LP in constructing a requirement plan.
Figure 9 shows the withdrawal patterns for this scenario.

The interesting features of Figure 9 are before the age of 70, when the RMD begins, and after the age of 86 when the RMD is in decline. During both periods Roth IRA withdrawals exceed IRA withdrawals. There are IRA withdrawals in every year to keep each year’s IRA withdrawals in lower tax brackets. Figure 9 indicates that IRA withdrawals are lower so as to delay some taxable withdrawals as long as possible to pay taxes as late as possible. After the age of 92 IRA withdrawals increase dramatically, but stay in the 10% tax-bracket, as the IRA is liquidated.

It may be noted that in the absence of other income, the IRA and Roth IRA lines are the mirror image of each other. This is because IRA withdrawals are either trying to minimize taxes or being constrained by the RMD. The Roth IRA is filling in the difference to support the spending plan.

This pattern remains the same when initial Roth IRA balances that are equal to or greater than IRA balances. Redoing this scenario with an estate requirement resulted in behavior similar to that discussed earlier in the Social Security scenario.

In other word, the more economical strategy is to delay some of the IRA withdrawals and accelerate some of the Roth IRA withdrawals. This is contrary to the conventional wisdom that IRA withdrawals should take precedence over Roth IRA withdrawals.

4.1.7 The IRA/After-tax Account Scenario

Table 7 shows the results of when there is no Roth IRA but there are significant beginning IRA ($580,000) and After-tax ($420,000) balances.
The proportions for the initial balances were computed using the same technique as for the three account, base scenario. Comparing the Efficiency column of Table 7 to Table 1, the base scenario, shows that having money in all three accounts offers some advantage over having no initial money in the Roth IRA.

Figure 10 shows the cash flow for this scenario across retirement.

In Figure 10 IRA to Roth IRA transfers dominate the early going. The plan begins with a large transfer followed by smaller transfer, in the interest of tax minimization, until the beginning of the RMD. At that point the large transfers continue for two more years. The RMD, which cannot be transferred, combined with the transfers creates a second spike.
The rest of the plan is pretty much as to be expected. When the After-tax account is depleted the Roth IRA takes up the slack at roughly the same level. While the Roth IRA is being distributed at a higher than expected rate the IRA is following the RMD down as the IRA is being depleted. Some of the IRA money is being retained to delay paying income taxes as long as possible, which accounts for the withdrawal adjustments at the end.

Figure 11 shows how IRA withdrawals are falling into the Federal income tax brackets.

![Figure 11: Tax Brackets for the IRA/After-tax Scenario](image)

The early spikes stay withing the 15% tax bracket, the same as when withdrawals are being forced by the RMD. There does not seem to be any reason for the IRA withdrawal spikes. The withdrawals late in the plan are at the top of the no tax bracket.

### 4.1.8 IRA Only Scenario

Even when the IRA is the only account containing money ORP still has to room to maneuver. The strategy is to move money into the other accounts early on and then use them to supplement IRA withdrawals later. The IRA Only Scenario begins with $1,000,000 in the IRA and zero balances in the other two accounts. Table 8 demonstrates:
Table 8: IRA Only Scenario

Table 9 shows the cash flow for the 5% investment return scenario. This is an example of transferring money from the IRA to the After-tax account as well as the Roth IRA.

Initially Roth IRA and the After-tax accounts are empty.

IRA to Roth IRA transfers (column IRA2Roth) begin at age 65. At age 70, when the RMD begins, IRA to Roth IRA transfers decrease because the RMD cannot be used to fund transfers to the Roth IRA. Instead the funds, which would have been sent to the Roth IRA, are transferred to the After-tax account (column Savings). At age 79 IRA withdrawals (column IRA) are reduced to the level of the RMD so that no additional transfers can be made to the Roth IRA and transfers to the After-Tax account end the
Retirement Saving Account Withdrawal Strategies

J. Welch  October 10, 2011

following year. The year after that distributions tied to the RMD decline as the IRA balance declines and the After-tax account (column After-tax) supplements IRA distributions to meet spending requirements.

At age 87 the After-tax account is depleted and withdrawals from the Roth IRA begin (column Roth IRA). In the final year, Age 95, the Roth IRA is depleted and one large, last withdrawal is made from the IRA to fund that year’s spending requirements and leave the estate at zero.

In order to level out taxes paid over time, money taken from the IRA before it is needed has to stored somewhere:

1. Roth IRA
2. After-tax account
3. The IRA itself, by keeping a limited amount in the IRA until the very end.

Table 9 is an excellent example of using all three storage places.

All IRA distributions stay inside the 15% tax bracket even the last big one.

4.1.9 Account Balance Scenarios

Most of the scenarios up to this point assumed beginning account balances are more or less of the same magnitude and that the IRA is the largest. Table 10 explores the models’ results for a range of IRA and Roth IRA beginning account balances. The scenarios are labeled according to the percentage of $1,000,000 that is in the account at the beginning. No allowance is made for taxes on the Roth IRA contributions. There is no initial After-tax balance.

<table>
<thead>
<tr>
<th>IRA</th>
<th>Roth IRA</th>
<th>CWS Spending</th>
<th>ORP Spending</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>90%</td>
<td>$44,300</td>
<td>$47,000</td>
<td>6%</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
<td>43,900</td>
<td>47,000</td>
<td>7</td>
</tr>
<tr>
<td>30</td>
<td>70</td>
<td>41,800</td>
<td>47,000</td>
<td>12</td>
</tr>
<tr>
<td>40</td>
<td>60</td>
<td>41,700</td>
<td>47,000</td>
<td>13</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>41,600</td>
<td>46,000</td>
<td>11</td>
</tr>
<tr>
<td>60</td>
<td>40</td>
<td>40,000</td>
<td>45,000</td>
<td>13</td>
</tr>
<tr>
<td>70</td>
<td>30</td>
<td>40,700</td>
<td>45,000</td>
<td>11</td>
</tr>
<tr>
<td>80</td>
<td>20</td>
<td>40,300</td>
<td>44,000</td>
<td>9</td>
</tr>
<tr>
<td>90</td>
<td>10</td>
<td>40,200</td>
<td>43,000</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 10: Extreme Balance Scenarios

The last table row (90/10) is probably typical of most retirees. The 60/40 row is similar to the IRA/Roth IRA scenario discussed above.

Figure 12 is a chart of the IRA distributions for the 10 scenarios in Table 10.
The legend labels the lines according to the percentage of the funds in the IRA for each scenario; the remainder are in the Roth IRA. For example the blue line at the bottom represents 10% IRA and 90% Roth IRA accounts.

All of the distribution lines possess the familiar pattern of the earlier scenarios. Before the RMD, withdrawals are made from the IRA for transfers to the Roth IRA except for the 10% (blue) line where no IRA distributions are made until the RMD begins. When the RMD starts the IRA distribution pegs to the RMD level until age 92 or so. Then the IRA distribution spikes when the funds being retained in the IRA for tax reasons are liquidated to bring the account balance to zero. Rather that take the last withdrawal into a higher tax bracket ORP will make oversized withdrawals for two or three years before the end of the plan. If, at age 92, the retiree discovers herself to be in good health she will have a cushion to for over the next few years thereby averting one of the retirement risks – outliving her income.

4.1.10 Summary

When all three accounts have the same ROR ORP will spread withdrawals from the IRA across the entire term to achieve lower taxes. ORP will try to push annual IRA distributions down into a low tax bracket while at the same time satisfying the RMD and maximizing total distributions by delaying IRA distributions as long as possible. Figure 2 points to the RMD as the important factor in setting IRA distribution levels. Except for the last few years IRA withdrawals are pegged to the RMD when it is active. The RMD and thus IRA withdrawals will be in a narrow range. Even when Social Security benefits are added to the mix in Figure 5 the IRA distributions are tied to the RMD.
4.2 Multiple Rates of Investment Return

Asset diversification is when each account contains a different asset class each with a different ROR. For example the IRA might contain growth stocks, the Roth IRA contains value stocks and the After-tax account contains bonds.

Table 11 shows scenarios where each account has its own ROR.

<table>
<thead>
<tr>
<th>ROR</th>
<th>CWS Spending</th>
<th>CWS Tx Rate</th>
<th>ORP Spending</th>
<th>ORP Tx Rate</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/5/5</td>
<td>$42,600</td>
<td>11%</td>
<td>$46,000</td>
<td>7%</td>
<td>8%</td>
</tr>
<tr>
<td>7/5/3</td>
<td>45,200</td>
<td>11</td>
<td>51,000</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>7/3/5</td>
<td>40,600</td>
<td>10</td>
<td>51,000</td>
<td>9</td>
<td>26</td>
</tr>
<tr>
<td>5/7/3</td>
<td>50,000</td>
<td>11</td>
<td>55,000</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>5/3/7</td>
<td>37,000</td>
<td>10</td>
<td>49,000</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>3/7/5</td>
<td>47,400</td>
<td>11</td>
<td>54,000</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>3/5/7</td>
<td>39,300</td>
<td>10</td>
<td>49,000</td>
<td>0</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 11: Investment Return Scenarios

The ROR column shows how the returns are assigned to the different accounts; IRA/Roth IRA/After-tax. For example the last row has the IRA returning 3%, the Roth IRA 5% and the After-Tax 7%. The first row is for the base scenario of Table 1 which has all three accounts with 5% RORs. The average ROR for the each scenario in Table 11 is 5%. Having three different ROR’s assigned to the three different accounts gives significantly improved spending as compared to the base case. This is because ORP has the freedom to exploit the differences of the RORs within the IRA tax situation. The ORP Tax Rate column was zero in the last four rows because the IRA was depleted by age 75.

The two scenarios with the highest spending rate have the highest ROR (7%) assets in the Roth IRA. The highest spending scenario is 5/7/3, i.e. rates of return of 5% for the IRA, 7% for the Roth IRA and 3% for the After-tax account.

Figure 11 shows the Withdrawal Report for the 5/7/3 scenario;
The After-tax account, with the lowest rate of return, is distributed early for spending. Since the IRA is returning 5% and the Roth IRA 7% there is an economic incentive to transfer the IRA to the Roth IRA all in the 15% tax bracket. This is done from age 65 to 69, while the After-tax account is being spent. From the ages of 70 to 79 the IRA and Roth IRA are distributed in parallel to satisfy spending. At age 80 the IRA is depleted, partly because of spending and partly because of transfers. Spending is satisfied with Roth IRA alone, with no taxes paid on withdrawals. When the Roth IRA has the highest ROR ORP sacrifices taxes on the early transfers to get the better Roth IRA ROR later on.

Having different RORs for the different accounts puts a new level of complexity into the model. Now not only is ORP trying to minimize taxes but it is also trying to maximize returns by moving money from the IRA to higher earning accounts. Restrictions in the tax code only allow for transfers from the IRA to the Roth IRA and After-tax accounts.

5 Conclusion

Some conclusions which can be drawn are:
1. Optimized withdrawal strategies from retirement saving accounts are superior to the conventional wisdom for every scenario. For most scenarios ORP outperforms CWS by 10% or more.
2. Withdrawing from retirement accounts in parallel, rather than serially, increases the amount of money available from retirement saving accounts for spending during retirement.
3. In many cases before the age of 90, IRA withdrawals are constrained by the RMD.
4. Tax-efficiency causes part of the withdrawals from the IRA to be delayed to the end of the term. IRA distributions may be lower and Roth IRA distributions higher than intuitive expectations.
5. Inflation is death to retirement spending.
6. Social Security benefits are important to withdrawal strategies because of their tax consequences and how they will affect withdrawal patterns.
7. Efficient withdrawal strategies are just as important to large retirement savings accounts as they are to modest accounts.
8. When there are no other sources of income, IRA and Roth IRA distributions are mirror images of each other.
9. Diversification pays; when accounts have different RORs the optimal plan may provide more spending than a plan computed for all accounts having the same ROR.
10. If the IRA ROR is significantly less than either of the other accounts then the efficient strategy is to make large transfers from the IRA to one of the other accounts at the start of retirement to take advantage of the higher ROR.
11. Some of the things that ORP does may seem arbitrary at first glance but upon examination they make economic sense.

In his paper about optimizing IRA and Roth IRA withdrawals under the progressive Federal Income tax structure Horan [10] arrives at the following withdrawal strategy: “… taking traditional IRA distributions up to the top of a ‘low’ tax bracket and satisfying the remainder of the withdrawal requirement from the Roth IRA yields residual accumulations that are substantially greater than the [conventional wisdom] strategy”.

In their paper about maximizing Social Security benefits by reducing personal income taxes Mahaney and Carlson [4] propose an IRA distribution strategy similar to Horan’s.

ORP’s results indicate that there are two dramatically different situations. If the IRA ROR is not less than that of the Roth IRA ROR then ORP partially confirms Horan’s strategy. Under this condition before age 70, while the RMD is inactive, and after age 90, when the RMD is no longer a factor, ORP follows Horan’s advice and limits withdrawals to the upper bound of the lowest possible tax bracket. In between the IRA withdrawals are constrained from below by the RMD whatever the tax bracket.

If the Roth IRA ROR is greater than the IRA ROR then ORP recommends partial rollovers of the IRA to the Roth IRA for the first few years of retirement even though the tax brackets are relatively high.

Horan goes on to recommend these “avenues for future research”:

1. “Mandatory distribution requirements from traditional IRAs.
3. “Integrate the impact on withdrawal location of taxable accounts outside traditional IRA and Roth IRA plans” (e.g. after-tax accounts, illiquid assets).

All of these avenues are explored in this paper.

ORP is not the only application of linear programming to optimizing withdrawals from retirement saving accounts. Two others are Ragsdale [17] in 1993 and Coopersmith [19] in 2011. Ragsdale modeled the penalties that applied to excessive withdrawals that were in the tax law at the time and have since been rescinded. Coopersmith reports that tax efficient models offer significant performance improvement over the common practice. Neither model is readily available to the public.

ORP offers these unique features:

- Instead of maximizing the estate, ORP maximizes annual spending, which is more meaningful to the individual user.
- ORP is a classic Operations Research model and system [23] running on an Internet server. ORP is probably the only linear programming application available for use by the general public over the Internet: www.i-orp.com.
- ORP was built using off-the-shelf commercial components rather than being a spreadsheet application on a desk top computer. (ORP’s matrix description language was implemented for the petroleum process industry [20] and its optimizer [21] was originally used for agricultural feed blending.)
- ORP models all three retirement account types, the RMD, the Federal progressive income tax and a variety of supplementary income streams.
- ORP models IRA rollovers to the Roth IRA and After-tax accounts.
- ORP models both the savings accumulation and distribution phases of retirement planning.
- ORP produces answers in a few seconds depending on the server load.

ORP is useful as a retirement planning tool for individual use because its parameter set can be tailored to an individual’s particular situation.

ORP will produce a wide variety of results for different circumstances. Factors such as pensions, Social Security benefits, selling illiquid assets, reverse mortgages and employment earnings during retirement will change ORP’s results, sometimes dramatically.

Trying to fit an individual’s situation into a set of published guidelines is inherently difficult and inefficient. It is more meaningful just to run the model.

6 Areas of Further Research

Section 4.2 and Appendix A touch on models where investment accounts have their own ROR different from the others. In both sections LP moved money into accounts with the higher ROR. The retiree may have an investment strategy that diversifies across asset classes and to have the LP chase after the
account with the highest ROR undoes the retiree’s diversification strategy. It may be desirable for the LP to maintain the ratio of balances in each account at the level of the initial values throughout retirement.

A second area of interest is to explore the consequences of relaxing the RMD.

**Appendix A**

Coopersmith and Sumutka’s [19] paper reports their experiments in applying LP to retirement planning. It is useful to compare the results of their system, called Tax-efficient or TE, with OPR’s for one of their reported runs:

The situation is a married couple, 64 years old, retiring at 65 with a $800,000 IRA and a $200,000 After-tax accounts. Their Social Security benefits are $20,000 annually, subject to inflation. Inflation is 2%. The ROR for their IRA is 5% and 7% for their After-tax count.

A comparison of the two models is shown in Table A.1.

<table>
<thead>
<tr>
<th></th>
<th>TE</th>
<th>ORP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spending</td>
<td>$60,000</td>
<td>$62,000</td>
</tr>
<tr>
<td>Estate</td>
<td>824,000</td>
<td>837,000</td>
</tr>
</tbody>
</table>

Table A.1: Results Comparison

TE sets the initial withdrawal to $60,000 and computes the estate. ORP does just the opposite setting the estate to $500,000 in today’s dollars and computing the initial withdrawal.

Considering that the tests were made with two entirely software systems these results are close enough to be credible.
Figure A.1 shows the withdrawal pattern for both systems

![Figure A.1: Withdrawal Reports](image)

Social Security benefits are not shown. They are the difference between Spending and total withdrawals.

The patterns of withdrawals are similar. Before age 70 ORP begins withdrawing more from the IRA than is needed for spending. The difference is transferred to the After-tax account. TE has After-tax account transfers also but they are not as noticeable in the figure.

The striking thing about both charts is that they are running counter to conventional wisdom. After-tax withdrawals begin well into retirement, not at the very beginning. After-tax withdrawals continue for the remainder of the plan in parallel with IRA withdrawals. This is caused by the After-tax ROR (7%) being significantly larger than the IRA ROR (5%). In both cases the LP system is sacrificing tax minimization in pursuit of higher returns.

7 References:

2. Benz, Christine: *How to Inflation-Adjust Your Retirement-Portfolio Withdrawals*,

3. Kitces, Michael E.: *Resolving the Paradox – Is the Safe Withdrawal Rate Sometimes Too Safe?*,


8. Welch, James: *The Optimal Retirement Planner* (ORP), ORP is available on the Internet at no charge [http://www.i-orp.com](http://www.i-orp.com)


13. Schlegel, Jeff: *Do you understand all the assumptions you are making regarding withdrawal rates?*, Financial Advisor Magazine, April, 2008.


